DEDUCTIVE SYSTEMS WITH UNIFIED MULTIPLE-CONCLUSION RULES

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Following the approach suggested by Brentano and accepted and developed by Łukasiewicz, we study the deductive systems that treat asserted and rejected propositions equally , in the same way.

By statement, we understand the expressions of form $\oplus A$ - "A being asserted", and $\ominus A$ - "A being rejected". Accordingly, by a consequence relation we understand a consequence relation between sets of statements and statements.

Unified deductive systems are the ordered pairs (Γ, R), where Γ is a set of axioms (statements of form $\oplus A$) and anti-axioms (statements of form $\ominus A$), while R is a set of multiple-conclusion rules – rules of form Δ/Ξ, where Δ and Ξ are the finite, perhaps empty, sets of statements. $\blacktriangledown$ denotes the empty set of premises, and $\blacktriangle$ denotes the empty set of conclusions.

Logic is coherent if the rule $\mathsf{Co} \coloneqq \oplus p, \ominus p/\blacktriangle$ is admissible in it; and logic is full if the rule $\mathsf{Fu} \coloneqq \blacktriangledown/\oplus p, \ominus p$ is admissible in it. The rules Co and Fu can be viewed as the rules expressing the law of non-contradiction and the law of the excluded middle.

Inclusion of the rules Co and Fu in a unified deductive system makes the system more flexible. In particular, adding these rules and the rule $\oplus\neg p/\ominus p$ to the classical propositional calculus (with postulated rule of substitution), results a unified deductive system that provides a complete Łukasiewicz style axiomatization: statement $\oplus A$ is derivable if and only if formula A is a classical theorem, while statement $\ominus A$ is derivable if and only if formula A is not a classical theorem.

Use of rules Co and Fu in formal derivations allows us to reduce multiple-conclusion rules to single-conclusion ones.

A unified deductive system that provides a complete and sound axiomatization of the classical logic with refutations is presented.