

*Исправления*  
*Erratum*

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**Erratum to: Natural Implication and Modus  
Ponens Principle**

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In the paper [1] some mistakes have occurred and the main results are corrected herein.

1. The theorem 1 on page 140 is not correct for matrix

$\mathfrak{M}_3 = \langle \{1, 1/2, 0\}, \rightarrow, \{1, 1/2\} \rangle$ , where  $\rightarrow$  is defined by truth-table:

$\rightarrow$	1	1/2	0
1	1	1	0
1/2	1	1	1/2
0	1	1	1

In particular, lemmas 1 and 2, which are used in the proof of the theorem 1, don't hold for the above mentioned matrix<sup>1</sup>. More over, we can give a counter-example, which shows that the rule *modus ponens* is not tautologousness-preserving in matrix  $\mathfrak{M}_3$ . For example,

$\vDash_{\mathfrak{M}_3} q \rightarrow ((q \rightarrow (q \rightarrow r)) \rightarrow r)$  and  
 $\vDash_{\mathfrak{M}_3} (q \rightarrow ((q \rightarrow (q \rightarrow r)) \rightarrow r)) \rightarrow ((p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)))$ ,  
but  $\not\vDash_{\mathfrak{M}_3} (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ .

2. The assertion that the extended class of natural implications consists of 80 implications is not correct. So, there are only 18 natural implications (and respectively logical matrices) for which the rule *modus ponens* is tautologousness-preserving, but not truth-preserving.

2 implications when  $D = \{1\}$ :

<sup>1</sup>The author is grateful to L. Devyatkin that he pointed out that fact.

$\rightarrow$	1	$1/2$	0
1	1	1	0
$1/2$	1	1	$a$
0	1	1	1

where  $a \in \{0, 1\}$ .

16 implications when  $D = \{1, 1/2\}$ :

$\rightarrow$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	$a$	$a$	$a$
0	1	1	1

$\rightarrow$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	$a$	$a$	$1/2$
0	1	$1/2$	1

$\rightarrow$	1	$1/2$	0
1	1	$b$	0
$1/2$	1	1	1
0	1	1	1

  

$\rightarrow$	1	$1/2$	0
1	1	0	0
$1/2$	$1/2$	$1/2$	$1/2$
0	1	1	1

$\rightarrow$	1	$1/2$	0
1	1	1	0
$1/2$	1	1	1
0	1	$1/2$	1

where  $a \in \{1, 1/2\}, b \in \{0, 1\}$ .

Thus, the extended class of natural implications consists of 48 implications (for 30 of which the rule *modus ponens* is truth-preserving).

### References

[1] Tomova, N.E. “Natural Implication and Modus Ponens Principle”, *Logical Investigations*, 2015, vol. 21, no 1, pp. 138–143.