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# A UNIVERSALLY ABNORMALITY-ADAPTIVE LOGIC[2]

**Abstract.** *The present paper presents a logic that allows for the abnormal behaviour of any logical constant and for the ambiguous behaviour of any non-logical constant, but nevertheless offers an interpretation of the premises that is as normal as possible.*

## 1. Aim of This Paper

Although adaptive logics are strictly formal logics, they integrate several typical aspects of 'argumentation': dynamic reasoning, meaning variance (of logical as well as non-logical constants), inferential information (as opposed to omniscience), and languages that are not compounded by pre-fixed building blocks (see especially [7] and [6]). The dynamics of the proofs relates to the fact that adaptive logics do *not*, as usual non-standard logics, *invalidate* certain rules of inference, but *restrict* their applications to consequences of the premises that fulfil certain conditions.

The first adaptive logics handled inconsistent sets of premises by interpreting them as consistently as possible (see, e.g., [4] and [1]). Later adaptive logics handled other forms of logical abnormality. Still later, logical abnormality deriving from the abnormal behaviour of non-logical constants (ambiguities) was integrated. Recently, the Ghent Group discovered adaptive logics that have nothing to do with logical abnormalities (compatibility, the consistent extensions of theories, abduction, …).

Considering abnormality-adaptive logics, one wonders whether it is possible to devise a logic that adapts to *all* forms of abnormalities. Such a system would open radically new perspectives on logic. It would allow for abnormalities of all kinds, but still presuppose (classical) normality 'unless and until proven otherwise'. In other words, it would be capable of recapturing all of Classical Logic – henceforth **CL** – while still allowing for all sorts of deviations from it. In this paper I present such a logic, **ACL∅2**.

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One way of describing the situation is by saying that, in **ACL∅2**, *all* derivation is conditional. That *A* is derivable from Γ does not only depend on the intended meaning of the logical constants and on the intended meaning stability of the non-logical constants; it is also depends on whether the specific content of Γ does not rule out that intended meaning and that intended stability.

## 2. Abnormality-Adaptive Logics

The first formulation of adaptive logics (see [4] and [1]) was proof-theoretical. Later a decent adequate semantics was developed (see [2], [8], and elsewhere). Meanwhile, many inconsistency-adaptive logics have been studied (see [19], [21], [22], [15], [17], and [12]), their use to several domains of application has been shown (see [3], [7], [16], and [18]), and several other logics (some non-monotonic logics, see [5] and [15], and all consequence relations defined from the Rescher-Manor mechanism, see [12]) have been integrated. I refer to [11] for a survey.

Even if the dynamics aspects of the proofs are among the most fascinating aspects of adaptive logics, space limitations prevent me to describe them here. So, let me at least mention one central feature.

At the predicative level, adaptive logics are not only undecidable[3]; for most of them, there is *no positive test* for derivability. Nevertheless, there are certain *criteria* that tell us, in specific cases, that a wff derived in a proof from Γ at a stage, is finally derived from Γ[4].

Where such criteria cannot be applied, it still can be demonstrated that derivability at a stage offers us the *best estimate* of the final consequences of the premises – best in view of the present understanding of the premises as revealed in the proof[5].

An abnormality-adaptive logic is defined from a *lower limit logic* (a monotonic paraconsistent logic) and an *upper limit logic* (usually **CL**) by an adaptive *strategy*. The latter determines the way in which the logic reacts to an inconsistency (or a disjunction of inconsistencies). All abnormality-adaptive logics interpret a theory \`as normaly as possible' - this phrase is ambiguous and is specified by the strategy.

The semantic characterization proceeds in terms of the models from the lower limit logic (that include the models from the upper limit logic). From those models of Γ a subset is selected in view of the

[3] The propositional fragments are just as decidable as that of **CL**.
[4] While derivability at a stage is a dynamic notion, final derivability is as static as any of the usual inference relations.
[5] Some proofs are \`smarter' than others. Yet, a proof as it stands reveals an understanding of the premises (see [6]).

strategy. The formulas true in all selected models are the adaptive consequences of $\Gamma$ (those finally derivable from it).

As an example, consider the semantics for **ACLuN2**. Let $\boldsymbol{L}$ be the language of **CL**, with $\boldsymbol{S}$, $\boldsymbol{C}$, $\boldsymbol{V}$, $\boldsymbol{P}^r$, $\boldsymbol{F}$ and $\boldsymbol{W}$ the sets of sentential letters, individual constants, individual variables, predicative letters of rank $r$, and formulas and wffs respectively. In order to handle the quantifiers in a simple way, we extend $\boldsymbol{L}$ to $\boldsymbol{L}^+$ by introducing a set of pseudo-constants $\boldsymbol{O}$ that has at least the cardinality of the largest models we want to consider. Let $\boldsymbol{W}^+$ be the set of pseudo-wffs of $\boldsymbol{L}^+$ and let $\sim\boldsymbol{W}^+ = \{\sim A \mid A \in \boldsymbol{W}^+\}$.

A model $M = \langle D, v\rangle$, in which $v$ is an interpretation of $\boldsymbol{W}^+$, and hence of $\boldsymbol{W}$, which is what we are interested in. The assignment function is defined by:

C1.1 $v : \boldsymbol{S} \rightarrow \{0, 1\}$

C1.2 $v : \boldsymbol{C}\cup\boldsymbol{O} \rightarrow D$ (where $D = \{v(\alpha) \mid \alpha \in \boldsymbol{C}\cup\boldsymbol{O}\}$)

C1.3 $v : \boldsymbol{P}^r \rightarrow \wp(D^r)$ (the power set of the $r$-th Cartesian product of $D$)

C1.4 $v : \sim\boldsymbol{W}^+ \rightarrow \{0, 1\}$

The valuation function $v_M$ determined by $M$ is defined as follows:

C2.1 $v_M : \boldsymbol{W}^+ \rightarrow \{0, 1\}$

C2.2 where $A \in \boldsymbol{S}$, $v_M(A) = v(\mathrm{A})$; $v_M(\bot) = 0$

C2.3 $v_M(\pi^r \alpha_1 \ldots \alpha_r) = 1$ iff $\langle v(\alpha_1), \ldots, v(\alpha_r)\rangle \in v(\pi^r)$

C2.4 $v_M(\alpha = \beta) = 1$ iff $v(\alpha) = v(\beta)$

C2.5 $v_M(\sim A) = 1$ iff $v_M(A) = 0$ or $v(\sim A) = 1$

C2.6 $v_M(A \supset B) = 1$ iff $v_M(A) = 0$ or $v_M(B) = 1$

C2.7 $v_M(A \wedge B) = 1$ iff $v_M(A) = 1$ and $v_M(B) = 1$

C2.8 $v_M(A \vee B) = 1$ iff $v_M(A) = 1$ or $v_M(B) = 1$

C2.9 $v_M(A \equiv B) = 1$ iff $v_M(A) = v_M(B)$

C2.10 $v_M((\forall\alpha) A(\alpha)) = 1$ iff $v_M(A(\beta)) = 1$ for all $\beta \in \boldsymbol{C}\cup\boldsymbol{O}$

C2.11 $v_M((\exists\alpha)A(\alpha))=1$ iff $v_M(A(\beta))=1$ for at least one $\beta \in \boldsymbol{C}\cup\boldsymbol{O}$

Truth in a model, semantic consequence, and validity are defined as usual.

The inconsistency-adaptive logic **ACLuN2** is obtained from **CLuN** by the Minimal Abnormality Strategy. The abnormal part of a **CLuN**-model is defined as $Ab(\mathrm{M}) =_{df} \{A \in \boldsymbol{F} \mid v_M(\exists(\mathrm{A} \wedge \sim\mathrm{A}) = 1\}$, where $\exists$ denotes a sequence of existential quantifiers over all variables free in $A$.

**Definition 1** *A* **CLuN**-model *M* of Γ is a *minimally abnormal* model of Γ iff no **CluN**-model *M'* of Γ is such that $Ab(M') \subset Ab(M)$.

**Definition 2** $\Gamma \models_{\mathbf{ACLuN2}} A$ iff *A* is true in all minimally abnormal models of Γ.

Apart from the Soundness and Strong Completeness of final derivability (see [8] for the definition) with respect to the semantics, many nice metaproperties have been proved. Proofs may be found in [8], [10], and some forthcoming papers. Some criteria for final derivability were obtained in view of tableau methods (see [13]), others in view of results of the block approach (see [6]).

The (negation-)incompleteness-adaptive logic that is the dual of **ACLuN2** is called **ACLaN2** and is obtained as follows. First we characterize **CLaN**, a logic allowing for g*a*ps rather than gl*u*ts with respect to *n*egation, by replacing in the semantics for **CLuN**:

C2.5 $v_M(\sim A) = 1$ iff $v_M(A) = 0$ *and* $v_M(\sim A) = 1$

**ACLaN2** is obtained from **CLaN** by the Minimal Abnormality Strategy, where $Ab(M) =_{df} \{A \in \boldsymbol{F} \mid v_M(\exists(\mathrm{A} \vee \sim\mathrm{A}) = 0\}$.

There are other logical abnormalities. Let us consider gaps with respect to conjunction, as they occur in the logic called (by the same naming convention) **CLaC**. Define $^{\wedge}\boldsymbol{W}^{+} = \{A \wedge B \mid A, B \in \boldsymbol{W}^{+}\}$, and accommodate the **CLuN**-semantics as follows. Clause C1.4 of the **CLuN**-semantics is replaced by

C1.4 $v : {}^{\wedge}\boldsymbol{W}^{+} \rightarrow \{0, 1\}$

whereas C2.5 and C2.7 now become:

C2.5 $v_M(\sim A) = 1$ iff $v_M(A) = 0$

C2.7 $v_M(A \wedge B) = 1$ iff $v_M(A) = v_M(B) = 1$ *and* $v(A \wedge B) = 1$

The adaptive logic **ACLaC2** is obtained from the lower limit logic **CLaC** by the Minimal Abnormality Strategy. We use classical conjunction (defined, *e.g.*, as $A \sqcap B =_{df} \sim(\sim A \vee \sim B)$) to define the abnormal part of a model *M*: $Ab(M) =_{df} \{A \wedge B \in \boldsymbol{F} \mid v_M(\exists(\mathrm{A} \sqcap B \sqcap \sim(A \wedge B))) = 1\}$, and adjust Definitions 1 and 2.

We may proceed similarly to allow for gluts or gaps with respect to other logical constants, and to devise a corresponding adaptive logic.

A somewhat different approach is to allow for *ambiguities* in the non-logical constants[6]. Here, the difference between the lower limit logic and the upper limit logic is obtained by a difference in *interpretation* of the premises. The upper limit logic interprets them in the usual way, but the lower limit logic interprets them by considering

[6] The first such adaptive logic taking is presented in [20]. The subsequent presentation differs in several respects from the line followed in that paper.

*each* occurrence of a non-logical constant as having a different meaning. This is indicated by a different index (a superscripted number) that is attached to each such occurrence in both the premises and the purported conclusion. In preparation of Section 3, I briefly spell this out.

The language $\boldsymbol{L}$ is upgraded in the obvious way to accommodate the indexed non-logical constants. Let the result be $\boldsymbol{L}^I$. Let $\boldsymbol{C}^I$ be the set of indexed individual constants. Let $\Gamma^\dagger$ and $A^\ddagger$ be the results of attaching a different index to each occurrence of a non-logical constant in $\Gamma\cup\{A\}$. For example $((p \wedge {\sim}q) \supset {\sim}p)^\dagger$ and ${\sim}p^\ddagger$ might be $((p^1 \wedge {\sim}q^1) \supset {\sim}p^2)$ and ${\sim}p^3$ respectively. The lower limit logic, **CLA**, is defined by[7]

$\Gamma \models_{\mathbf{CLA}} A$ iff $\Gamma^\dagger \models_{\mathbf{CL}} A^\ddagger$

Obviously $(p\wedge q) \not\models_{\mathbf{CLA}} p$, as nothing warrants that $v_M(p^1) = v_M(p^2)$. However, ${\sim}(p^1{\equiv}p^2)$ counts as an *abnormality* with respect to the normal interpretation of the premises. For $\boldsymbol{C}$ the abnormalities are of the form ${\sim}\alpha^i{=}\alpha^j$, for $\boldsymbol{P}^r$ they are of the form ${\sim}((\forall\alpha_1)\ldots(\forall\alpha_r)(\pi^i\, \alpha_1 \ldots \alpha_r \equiv \pi^j\, \alpha_1 \ldots \alpha_r))$. Define *Ab*(*M*) accordingly. By selecting the minimally abnormal models of $\Gamma$, the adaptive logic **ACLA2** interprets the premises as normally (that is, unambiguously) as possible.

It is tempting to devise logics that are adaptive with respect to several kinds of abnormalities, for example with respect to inconsistency and incompleteness, or with respect to ambiguities and gluts with respect to the universal quantifier. Two brief remarks will clarify how this is done.

A logic that allows for both gluts and gaps with respect to, for example, negation – we call this logic **CLoN** – is obtained by replacing, in the **CLuN** semantics, clause C2.5 as follows:

C2.5 $v_M({\sim}A) = v({\sim}A)$

With this clarified, the combination of any logical abnormalities is fairly obvious (and so is the definition of *Ab*(*M*)).

If we combine many abnormalities, the remaining classical connectives might not be functionally complete, which will hinder the definition of *Ab*(*M*). As I need to consider such a case in the subsequent section, I briefly discuss the matter. Let us again consider $\{A \in \boldsymbol{F} \mid v_M(\exists(A \wedge {\sim}A) = 1\}$ and eliminate the "$\exists$" and "$\wedge$" from it. Suppose that $x_1,\ldots, x_n$ are the variables that occur free in $A$. Let $[A]^{o1,\ldots,on}_{x1,\ldots,xn}$ be the result of systematically replacing each $x_i$ by some $o_i \in \boldsymbol{C}\cup\boldsymbol{O}$. To

[7] In all subsequent examples, I only consider the case where the *i*-th occurrence of some non-logical constant receives index *i*. All other ways of indexing obviously deliver isomorphic results, as long as the choice fulfils the above convention on $\Gamma^\dagger$ and $A^\ddagger$.

avoid clutter, I henceforth write **x** for $x_1, \ldots, x_n$ and **o** for $o_1, \ldots, o_n$. Remark that $v_M(\exists(A \wedge {\sim}A) = 1$ iff there are **o** such that $v_M([A]^o_x) = v_M([{\sim}A]^o_x) = 1$[8]. Proceeding thus, we may eliminate all 'auxiliary' logical constants from the definition of *Ab*(*M*).

## 3. The Empty Logic and Its Adaptive Extension

Here I combine all kinds of abnormalities. The lower limit logic will be called **CL∅**. We start with **CLoX**, the logic that allows both gluts and gaps with respect to all logical constants. Where $M = \langle D, v\rangle$ is a **CLoX**-model, $v$ is defined by:

C1.1 $v : \boldsymbol{S} \rightarrow \{0, 1\}$

C1.2 $v : \boldsymbol{C} \cup \boldsymbol{O} \rightarrow D$ (where $D = \{v(\alpha) \mid \alpha \in \boldsymbol{C} \cup \boldsymbol{O}\}$)

C1.3 $v : \boldsymbol{P}^r \rightarrow \wp(D^r)$ (the power set of the $r$-th Cartesian product of $D$)

C1.4 $v : {}^{=}\boldsymbol{W}^{+} \rightarrow \{0, 1\}$

C1.5 $v : {}^{\sim}\boldsymbol{W}^{+} \rightarrow \{0, 1\}$

C1.6 $v : {}^{\supset}\boldsymbol{W}^{+} \rightarrow \{0, 1\}$

C1.7 $v : {}^{\wedge}\boldsymbol{W}^{+} \rightarrow \{0, 1\}$

C1.8 $v : {}^{\vee}\boldsymbol{W}^{+} \rightarrow \{0, 1\}$

C1.9 $v : {}^{\equiv}\boldsymbol{W}^{+} \rightarrow \{0, 1\}$

C1.10 $v : {}^{\forall}\boldsymbol{W}^{+} \rightarrow \{0, 1\}$

C1.11 $v : {}^{\exists}\boldsymbol{W}^{+} \rightarrow \{0, 1\}$

The valuation function $v_M$ determined by $M$ is defined as follows:

C2.1 $v_M : \boldsymbol{W}^{+} \rightarrow \{0, 1\}$

C2.2 where $A \in \boldsymbol{S}$, $v_M(A) = v(A)$; $v_M(\bot) = 0$

C2.3 $v_M(\pi^r \alpha_1 \ldots \alpha_r) = 1$ iff $\langle v(\alpha_1), \ldots, v(\alpha_r)\rangle \in v(\pi^r)$

C2.4 $v_M(\alpha = \beta) = 1$ iff $v(\alpha = \beta)$

C2.5 $v_M({\sim}A) = v({\sim}A)$

C2.6 $v_M(A \supset B) = v(A \supset B)$

C2.7 $v_M(A \wedge B) = v(A \wedge B)$

C2.8 $v_M(A \vee B) = v(A \vee B)$

C2.9 $v_M(A \equiv B) = v(A \equiv B)$

C2.10 $v_M((\forall\alpha) A(\alpha)) = v((\forall\alpha) A(\alpha))$

C2.11 $v_M((\exists\alpha)A(\alpha)) = v((\exists\alpha)A(\alpha))$

[8] Obviously $[{\sim}A]^o_x = [{\sim}A]^o_x$. If expressions such as $[A \supset B]^o_x$ and $[A]^o_x$ are used in the same clause, **x** always refers to the variables free in the longest formula.

Truth in a model, semantic consequence, and validity are defined as usual.

We now upgrade **CLoX** to **CL∅**. Let $\Gamma^{\dagger}$ and $A^{\ddagger}$ be as in Section 2.

**Definition 3** $\Gamma \models_{\mathbf{CL\varnothing}} A$ iff $\Gamma^{\dagger} \models_{\mathbf{CLoX}} A^{\ddagger}$

$\Gamma \models_{\mathbf{CL\varnothing}} A$ iff $\bot \in \Gamma$. So, if we remove $\bot$ from $\boldsymbol{L}$, $\Gamma \not\models_{\mathbf{CL\varnothing}} A$ for *all* $\Gamma$ and $A$.

To formulate **ACL∅2**, obtained from **CL∅** by the Minimal Abnormality strategy, I define $Ab(M)$ as the union of the sets listed below (defined in terms of **CLoX**-models). I suppose throughout that $A$, $B \in \boldsymbol{F}$ and that each defining clause is preceded by: for some $\mathbf{o} \in \boldsymbol{C}^{I} \cup \boldsymbol{O}$.

$\{A \mid v_M([A]^{o}_{x}) = v_M([{\sim}A]^{o}_{x})\}$,

$\{\langle A \supset B, A, B\rangle \mid v_M([A \supset B]^{o}_{x}) = v_M([A]^{o}_{x}) = 1$ and $v_M([B]^{o}_{x}) = 0\}$,

$\{\langle A \supset B, A\rangle \mid v_M([A \supset B]^{o}_{x}) = 0$ and $v_M([A]^{o}_{x}) = 0\}$,

$\{\langle A \supset B, B\rangle \mid v_M([A \supset B]^{o}_{x}) = 0$ and $v_M([B]^{o}_{x}) = 1\}$,

$\{\langle A \wedge B, A, B\rangle \mid v_M([A \wedge B]^{o}_{x}) = 0$ and $v_M([A]^{o}_{x}) = v_M([B]^{o}_{x}) = 1\}$,

$\{\langle A \wedge B, A\rangle \mid v_M([A \wedge B]^{o}_{x}) = 1$ and $v_M([A]^{o}_{x}) = 0\}$,

$\{\langle A \wedge B, B\rangle \mid v_M([A \wedge B]^{o}_{x}) = 1$ and $v_M([B]^{o}_{x}) = 0\}$,

$\{\langle A \vee B, A, B\rangle \mid v_M([A \vee B]^{o}_{x}) = 1$ and $v_M([A]^{o}_{x}) = v_M([B]^{o}_{x}) = 0\}$,

$\{\langle A \vee B, A\rangle \mid v_M([A \vee B]^{o}_{x}) = 0$ and $v_M([A]^{o}_{x}) = 1\}$,

$\{\langle A \vee B, B\rangle \mid v_M([A \vee B]^{o}_{x}) = 0$ and $v_M([B]^{o}_{x}) = 1\}$,

$\{\langle A \equiv B, A, B\rangle \mid v_M([A \equiv B]^{o}_{x}) \neq (v_M([A]^{o}_{x}) = v_M([B]^{o}_{x}))\}$[9],

$\{\langle (\forall\alpha)\, A(\alpha), A(\beta)\rangle \mid \beta \in \boldsymbol{C}^{I};\ v_M([(\forall\alpha)\, A(\alpha)]^{o}_{x}) = 1$ and $v_M([A(\beta)]^{o}_{x}) = 0\}$,

$\{\langle (\forall\alpha)\, A(\alpha), A(\beta)\rangle \mid \beta \in \boldsymbol{O};\ v_M([(\forall\alpha)\, A(\alpha)]^{o}_{x}) = 1$ and $v_M([A(\gamma)]^{o}_{x}) = 0$ for some $\gamma \in \boldsymbol{C}^{I} \cup \boldsymbol{O}\}$,

$\{\langle (\forall\alpha)\, A(\alpha), A(\beta)\rangle \mid \beta \in \boldsymbol{V};\ v_M([(\forall\alpha)\, A(\alpha)]^{o}_{x}) = 0$ and $v_M([A(\gamma)]^{o}_{x}) = 1$ for all $\gamma \in \boldsymbol{C}^{I} \cup \boldsymbol{O}\}$,

$\{\langle (\exists\alpha)\, A(\alpha), A(\beta)\rangle \mid \beta \in \boldsymbol{V};\ v_M([(\exists\alpha)\, A(\alpha)]^{o}_{x}) = 1$ and $v_M([A(\gamma)]^{o}_{x}) = 0$ for all $\gamma \in \boldsymbol{C}^{I} \cup \boldsymbol{O}\}$,

$\{\langle (\exists\alpha)\, A(\alpha), A(\beta)\rangle \mid \beta \in \boldsymbol{C}^{I};\ v_M([(\exists\alpha)\, A(\alpha)]^{o}_{x}) = 0$ and $v_M([A(\beta)]^{o}_{x}) = 1\}$,

$\{\langle (\exists\alpha)\, A(\alpha), A(\beta)\rangle \mid \beta \in \boldsymbol{V};\ v_M([(\exists\alpha)\, A(\alpha)]^{o}_{x}) = 0$ and $v_M([A(\gamma)]^{o}_{x}) = 1$ for some $\gamma \in \boldsymbol{C}^{I} \cup \boldsymbol{O}\}$,

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[9] The condition obviously abbreviates that $v_M([A \equiv B]^{o}_{x})$ is 1 (respectively 0) whereas $v_M([A]^{o}_{x})$ is not (respectively is) identical to $v_M([B]^{o}_{x})$. Similarly for "=" below.

$\{<\alpha, \beta> \mid \alpha, \beta \in \boldsymbol{C}^{\mathrm{I}}; v_M(\alpha = \beta) \neq (v(\alpha) = v(\beta))\}$,
$\{<\alpha, \beta> \mid \alpha \in \boldsymbol{C}^{\mathrm{I}}; \beta \in \boldsymbol{V}$; for some $\gamma \in \boldsymbol{C}^{\boldsymbol{I}} \cup \boldsymbol{O}$, $v_M(\alpha = \gamma) \neq (v(\alpha) = v(\gamma))\}$,
$\{<\alpha, \beta> \mid \alpha \in \boldsymbol{V}; \beta \in \boldsymbol{C}^{\mathrm{I}}$; for some $\gamma \in \boldsymbol{C}^{\boldsymbol{I}} \cup \boldsymbol{O}$, $v_M(\gamma = \beta) \neq (v(\gamma) = v(\beta))\}$,
$\{<\alpha, \beta> \mid \alpha, \beta \in \boldsymbol{V}$; for some $\gamma, \delta \in \boldsymbol{C}^{\boldsymbol{I}} \cup \boldsymbol{O}$, $v_M(\gamma = \delta) \neq (v(\gamma) = v(\delta))\}$,
$\{<A, i, j> \mid A \in \boldsymbol{S}$ and $v_M(A^i) \neq v_M(A^j)\}$,
$\{<\alpha, i, j> \mid \alpha \in \boldsymbol{C}$ and $v(\alpha^i) \neq v(\alpha^j)\}$, and
$\{<\pi, i, j> \mid \pi \in \boldsymbol{P}^r$ and $v(\pi^i) \neq v(\pi^j)\}$,

**Definition 4** *A* **CLoX**-model *M* of $\Gamma^\dagger$ is a *minimally abnormal* model of $\Gamma^\dagger$ iff no **CLoX**-model *M′* of $\Gamma^\dagger$ is such that $Ab(M') \supset Ab(M)$.

**Definition 5** $\Gamma \models_{\mathbf{ACL\varnothing 2}} A$ iff $A^\ddagger$ is true in all minimally abnormal models of $\Gamma^\dagger$.

It is easily provable (compare [8]) that, if $\Gamma$ has **CL**-models, then the minimally abnormal models of $\Gamma^\dagger$ are those in which all logical constants as well as all occurrences of non-logical constants behave normally. Hence:
$\Gamma \models_{\mathbf{ACL\varnothing 2}} A$ iff $\Gamma \models_{\mathbf{CL}} A$.

If $\Gamma$ has *no* **CL**-models, then (except for border cases) the minimally abnormal models of $\Gamma^\dagger$ are a proper subset of the **CLoX**-models of $\Gamma^\dagger$. In this case, **ACL∅2** still delivers a minimally abnormal interpretation of $\Gamma$: all **CL**-consequences of $\Gamma$, except for those that do not follow from $\Gamma$ in view of the disjunctions of abnormalities that are verified by *all* **CLox**-models of $\Gamma^\dagger$.

I leave it to the reader to check the following properties of **ACL∅2**:

$p, {\sim}p \not\models_{\mathbf{ACL\varnothing 2}} q$

$p \wedge q, {\sim}p \not\models_{\mathbf{ACL\varnothing 2}} q$

$p \wedge {\sim}r, {\sim}p \wedge (q \supset r) \not\models_{\mathbf{ACL\varnothing 2}} {\sim}q$

$\forall x(Px \supset Qx), Pa, {\sim}Qa, Pb, {\sim}Qc \not\models_{\mathbf{ACL\varnothing 2}} Qb \wedge {\sim}Pc$

Remark that $p, {\sim}p \not\models_{\mathbf{ACL\varnothing 2}} p$ and $p, {\sim}p \not\models_{\mathbf{ACL\varnothing 2}} {\sim}p$. Some of the minimally abnormal interpretations of the premises requires the non-logical constant (represented here by the propositional letter) *p* to behave ambiguously: the truth-value of $p^1$ is different from that of $p^2$. So, the truth-value $p^3$ is bound to agree with that of either $p^1$ or $p^2$. As the same reasoning applies to the other non-logical constants, one easily proves the remarkable:

**Theorem 1** For all $\Gamma$, $\mathrm{Cn}_{\mathbf{ACL\varnothing 2}}(\Gamma)$ is consistent.

## 4. In Conclusion

As promised, **ACL∅2** is a universally abnormality-adaptive logic. Allowing all kinds of abnormalities in the premises, it still interpretats them “as normal as possible”. The latter expression has (as always) a specific meaning. But this meaning is an interesting one: whatever is consistently derivable from the premises. As one cannot determine beforehand which abnormalities will or may occur in the theory, one cannot justify beforehand the choice of an abnormality-adaptive logic. **ACL∅2** removes this weakness.

Let me remind the reader that the aim of abnormality-adaptive logics is not offer the `final' interpretation of the premises. Logical abnormalities will have to be ruled out; defective theories have to be replaced. This replacement is not a matter of logic, but it *requires* the logical analysis of the defective theory. *This* is what an adaptive logic should provide, and this is what **ACL∅2** actually does provide for a very broad set of contexts.

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