Between $\text{Int}_{<\omega,\omega>}$ and intuitionistic propositional logic[1]

VLADIMIR M. POPOV

ABSTRACT. This short paper presents a new domain of logical investigations.

Keywords: paralogic, paracomplete logic, paraconsistent logic, paranormal logic, intuitionistic propositional logic

The language L of each logic in the paper is a standard propositional language whose alphabet is as follows: $\{\&, \vee, \supset, \neg, (,), p_1, p_2, p_3, \ldots\}$. As it is expected, &, $\vee$, $\supset$ are binary logical connectives in L, $\neg$ is a unary logical connective in L, brackets (,) are technical symbols in L and $p_1, p_2, p_3, \ldots$ are propositional variables in L. A definition of L-formula is as usual. Below, we say 'formula' instead of 'L-formula' only and adopt the convention on omitting brackets. A formula is said to be quasi-elemental iff no logical connective in L other than $\neg$ occurs in it. A length of a formula A is, traditionally, said to be the number of all occurrences of the logical connectives in L in A. A logic is said to be a non-empty set of formulas closed under the rule of modus ponens in L and the rule of substitution of a formula into a formula instead of a propositional variable in L.

Let us agree that α and β are arbitrary elements in $\{0, 1, 2, 3, \ldots \omega\}$. We define calculus $\text{HInt}_{<\alpha,\beta>}$. This calculus is a Hilbert-type calculus, the language of $\text{HInt}_{<\alpha,\beta>}$ is L. $\text{HInt}_{<\alpha,\beta>}$ has the rule of modus ponens in L as the only rule of inference. The notion of a proof in $\text{HInt}_{<\alpha,\beta>}$ and the notion of a formula provable

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in this calculus are defined as usual. Now we only need to define the set of axioms of $\mathrm{HInt}_{<\alpha,\beta>}$.

A formula belongs to the set of axioms of calculus $\mathrm{HInt}_{<\alpha,\beta>}$ iff it is one of the following forms (A, B, C denote formulas):

(I) $(A \supset B) \supset ((B \supset C) \supset (A \supset C))$, (II) $A \supset (A \vee B)$, (III) $B \supset (A \vee B)$, (IV) $(A \supset C) \supset ((B \supset C) \supset ((A \vee B) \supset C))$, (V) $(A\&B) \supset A$, (VI) $(A\&B) \supset B$, (VII) $(C \supset A) \supset ((C \supset B) \supset (C \supset (A\&B)))$, (VIII) $(A \supset (B \supset C)) \supset ((A\&B) \supset C)$, (IX) $((A\&B) \supset C) \supset (A \supset (B \supset C))$, (X, α) $\neg D \supset (D \supset A)$, where D is formula which is not a quasi-elemental formula of a length less than α, (XI, β) $(E \supset \neg(B \supset B)) \supset \neg E$, where E is formula which is not a quasi-elemental formula of a length less than β.

Let us agree that, for any j and k in $\{0, 1, 2, 3, \ldots \omega\}$, $\mathrm{Int}_{<j,k>}$ is the set of formulas provable in $\mathrm{HInt}_{<j,k>}$. It is clear that, for any j and k in $\{0, 1, 2, 3, \ldots \omega\}$, a set $\mathrm{Int}_{<j,k>}$ is a logic. It is proved that $\mathrm{Int}_{<0,0>}$ is the set of intuitionistic tautologies in L (that is, the intuitionistic propositional logic in L). By S we denote the set of all logics which include logic $\mathrm{Int}_{<\omega,\omega>}$ and are included in $\mathrm{Int}_{<0,0>}$ and by *ParaInt* we denote $S \setminus \{\mathrm{Int}_{<0,0>}\}$. Note logic $\mathrm{Int}_{<\omega,\omega>}$ is the intersection of all logics, other than itself, in *ParaInt.* The set *ParaInt* is of interest for scholars who study paralogics (paraconsistent or paracomplete logics). The set *ParaInt* contains (1) a continuous set of paraconsistent, but non-paracomplete logics, (2) a continuous set of paracomplete, but non-paraconsistent logics, (3) a continuous set of paranormal logics. We have some results concerning both logics from *ParaInt* and classes of such logics. In particular, we have methods to construct axiomatisations (sequent calculus and analytic-tableaux calculus) and semantics (in the sense of Kripke) for any logic $\mathrm{Int}_{<j,k>}$, where j and k in $\{0, 1, 2, 3, \ldots \omega\}$.

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