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# NESTED SUPERVALUATIONS FOR FUTURE INDEFINITE CONTINGENT VAGUENESS

**Abstract.** *It seems that some English sentences naturally invite irreducibly multiple application of the method of supervaluations. In this paper, I use the machinery and ideology of Game-Theoretical Semantics (GTS)[1] to investigate this phenomenon and trace some of its implications, both on the logical and linguistic sides.*

## 1. Supervaluational chance-prenexed games for formal languages

Supervaluational semantic games[2] are chance-prenexed, that is, there is exactly one occurrence of chance move in the game-tree, and this occurrence opens the whole game.

Chance-prenexedness of supervaluational games seems crucial for preservation of classical logical truths by the method of supervaluations. Game-theoretically speaking, classicality is secured by the fact that all the subgames that are immediately governed by the initial chance move are standard classical semantic games; given any classical logical truth $\tau$ and any reference-point $\alpha$, such a game, $\Gamma(\tau,\alpha)$, would necessarily possess at least one winning strategy for the Verifier - that much is guaranteed (i) by the classicality of each immediate subgame of $\Gamma(\tau,\alpha)$ (in particular, by the absence of chance moves from it), and (ii) by the fact that classical games yield the same bivalent results as standard Tarski-type recursive semantics. But if, in the structure of the game, there happen to be some further, non-initial occurrences of chance moves, then those occurrences would be bound to occur within immediate subgames, which could destroy the guarantee of there being any winning strategies for the Verifier.

On the first glance, though, it seems that we have no reasons to worry about such a possibility - it seems to have been ruled out, at least for such formal supervaluational languages as Thomason's[3] $L_T$, by a remarkable rule that governs construction of semantic games for such languages. Take two arbitrary sentences of $L_T$, A and B, and form their

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[1] Detailed accounts of GTS are available, e.g., in Hintikka and Sandu (1997), Hintikka and Sandu (1991, pp.15-37), Hintikka and Kulas (1983, pp.1-31), and Hintikka's contributions to Saarinen (1979).
[2] Cf. Blinov (1994).
[3] For further details on $L_T$, see Thomason (1970).

disjunction, A ∨ B. To construct the supervaluational semantic game for A ∨ B, relative to a reference-point α, on the basis of games for A and B, one should proceed in a very special way: one should, first, remove the initial chance moves from both Γ(A; α) and Γ(B; α); second, for any history $h \in H_\alpha$, construct the classical Γ ∨ subgame Γ(A ∨ B; α, $h$) on the basis of two classical subgames Γ(A; α, $h$) and Γ(A; α, $h$); third, re-attach the initial chance move to the family of newly constructed classical subgames of the form Γ(A ∨ B; α,$h$). One should, so to say, first, uncork the semantic bottles of Γ(A; α) and Γ(B; α), second, mix their contents in the disjunctional way, and then re-cork the new bigger bottle of the family of Γ(A ∨ B; α,$h$) by the chance-move cork again. For the sake of a technical label, I will dub this procedure the *Uncork-Re-Cork Rule* of construction of supervaluational semantic games.

Thus, it is exactly the Uncork-Re-Cork Rule that secures the uniqueness and initiality of the chance move in supervaluational games, that is, their *chance-prenexedness*. In the absence of such special construction rule, we would proceed in a way that is usual for standard, non-supervaluational languages, namely, we would take Γ(A; α) and Γ(B; α) and merge them into one new semantic game by simply hooking the game-trees of both Γ(A; α) and Γ(B; α) on to the two respective nodes of the initial disjunction move. Of course, the resulting game, Γ*(A ∨ B; α), would *not* be chance-prenexed. There would be *two* occurrences of chance moves in it: one in the subgame Γ(A; α), another, in the subgame Γ(B; α). Both would be *non-initial*. The initial move would be disjunctional, that is, played by the Verifier.

Suppose now that B is ¬ A. There is no guarantee whatsoever that the Verifier has a winning strategy in Γ*(A ∨ ¬ A; α). In fact, it is easy to see that in a case when both A and ¬ A are (super)truth-value gaps at α, the Verifier has no winning strategies in Γ*(A ∨ ¬ A; α). Thus, there is no preservation of classical logical truths by supervaluational games without the Uncork-Re-Cork Construction Rule.

## 2. Formal language $L_T$: Game rule for chance moves

A construction of a semantic game for a formal-language sentence is determined by the totality of game rules for that language.

For example, if a sentence S has the form $S_1$ & $S_2$, the construction of Γ(S) begins with an application of (G.&), the game rule for &:

(G.&)  If the game has reached a sentence of the form $F_1$ & $F_2$, then the Falsifier chooses $F_i$ (i = 1 or 2). Then the game is continued with respect to the chosen conjunct $F_i$.

One striking dissimilarity of the chance-move game rule, as compared with (G.&) and its like, is that information about the form of the input sentence which has been the crucial part of the application conditions for the latter rule is just irrelevant for the former: the form can be any. But if it is not the form of the input sentence that triggers the application of the chance-move rule, then what does? It seems that at this stage one can come up with two different answers:

*Answer 1: Rigid position approach.* The two crucial facts about chance moves in semantic games for $L_T$ are that, for any sentence S in $L_T$, (i) there is exactly one chance move in the semantic game $\Gamma(S)$; (ii) the unique chance move is positionally rigid – it always opens the game. That much is pre-determined by the standard interpretation of supervaluations for $L_T$, which, in turn, results from our intention to maintain the classicality of the logic of $L_T$. Thus, the application of the chance-move rule for $L_T$ should be triggered by (i) and (ii). That is, the application conditions of the chance-move game rule should be something like this: 'A semantic game for a sentence S that is to be evaluated on its own right (not as a subformula of some other sentence) always contains exactly one chance move, and that chance move always opens the game'[4].

*Answer 2: Reference point approach.* The method of supervaluations is inherently connected with evaluation of sentences at *incomplete* models. The main function of supervaluational chance moves consists in resolving the incompleteness of the model which constitutes the input reference point for the sentence under evaluation. Correspondingly, the application of the chance-move rule should be triggered by the nature of the input reference-point: 'Whenever the input reference-point is an incomplete model *m*, the Chance should resolve the whole of the relevant incompleteness by choosing one of the complete (or, more generally, less incomplete) models that participate in *m*.' Since, as far as $L_T$ is concerned, the only incomplete models that can be input reference-points are times, we can be even more specific: 'Whenever the input reference-point is a time $\alpha$, the Chance should resolve the incompleteness of $\alpha$, as instantiated by $H_\alpha$, by choosing a history from $H_\alpha$.'

Note that even though the Answers 1 and 2 use two different methods of determining the application conditions of the chance-move rule, still when applied to such a language as $L_T$, the results of the two methods coincide, namely, both determine that in a semantic game for a sentence of $L_T$ there should be exactly one chance move, and it should open the game.

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[4] Cf. the statement of the game rule (G.*Init*) in Blinov (1994, p.323).

### 3. Natural languages: Game rules and ordering principles

The process of constructing the semantic game associated with a natural-language sentence S is governed by (i) game rules; (ii) ordering principles. Roughly speaking, the role of game rules in the game construction is to assign game moves to various lexical items occurring in S, while the role of ordering principles is to determine the order of application of game rules.

Summing up, the game rules for natural languages are *lexicalist* in that their application is triggered by a *lexical item* occurring in the sentence at issue, while the order of application of various rules is governed by the ordering principles.

Need the chance-move rule for a natural language be an exception to the general lexicalist approach to game-rules formation? Need, that is, its application conditions for a language like English remain insensitive to the lexical composition of the sentence at issue? I believe they need not. It is, I think, a *prima facie* plausible conjecture that if we assess an English sentence as calling for the method of supervaluations, then, more often than not, we are able to locate the lexical source of the call.

It is, most graphically, the case with vagueness as treated supervaluationally: vague predicates and names are represented, in a sentence, by so many occurrences of the corresponding words. Another lexicalist-friendly example is future contingents. Their natural English marker is the lexical item 'will'[5]. Here is the questionnaire format for a supervaluational chance-move game rule for a fragment of English in which the sources of supervaluations are limited to vagueness and future contingents.

| A | Application conditions | 1 | The form of the input sentence | 'X - Y -W' where 'Y' is an occurrence of a supervaluational word or phrase, that is, a vague predicate or name, or else the occurrence of 'will' |
| --- | --- | --- | --- | --- |
|  |  | 2 | The format of the input reference point | time $\alpha$ |

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[5] Admittedly, the lexical and/or morphological markers of the past and present tense call for supervaluations just as much. The difference, though, is that, assuming that the only branching is toward the future, supervaluating a (purely) past-tensed or present-tensed sentence is bound to be a trivial procedure in that all the constituent bivalent valuations would collapse into one.

| B | Prescriptive content of the rule | 1 | The player who performs the move | the Chance |
|---|---|---|---|---|
| | | 2 | What the player does to perform the move | He picks up a precisification of time $\alpha$, as determined by the item Y, or else a history *h* from $H_\alpha$, according to whether the triggering word Y is a vague lexical unit or the word 'will' |
| C | Output results | 1 | The form of the output sentence | same as the input sentence |
| | | 2 | The output reference point | a precisification of time $\alpha$ or else $\langle \alpha, h \rangle$ |

Figure 1

*Comment 1*. The most salient feature of the lexicalist chance-move rule, is that the supervaluational incompleteness of the input reference-point is to be resolved step by step rather that at one initial swoop. The number of steps equals to the number of applications of the chance-move rule, which in turn depends on the number of the occurrences of chance-move-triggering lexical items in the sentence at issue.

*Comment 2*. Obviously, we should provide, in addition to the above game rule, some ordering principles that would endow the applications of the chance-move rule with an intuitively justifiable degree of priority before the applications of the rest of the game rules. Ideally, all the applications of the chance-move rule are to precede the applications of other rules. Such an order would perfectly correspond to the nature of the method of supervaluations as usually conceived by its practitioners. But we have no *a priori* guarantee that such an ideal is compatible with the totality of independent evidence on the semantic structure of English sentences. In fact, I will be arguing for their incompatibility. So, if my argument is correct, it would imply that the above-mentioned ideal stands in need of some qualifications.

## 4. Testing the three approaches:
## Future indefinite contingent vagueness

Now, my main point concerning the chance-move rule for natural languages is that neither the Answer 1 nor the Answer 2 (cf. Section 2 above) will do; each of them is bound to clash with some basic seman-

tic intuitions of ours. We should adopt, instead, the lexicalist approach to fixing the chance-move rule, which, thus, need not be an exception to the overall spirit of GTS for natural languages.

Consider sentence (1):

(1)  Sometimes in the future, the number of trees in Brisbane will be even.

The adverbial phrase 'sometimes in the future' determines an existential quantification over times. The vagueness of Brisbane's boundaries is contingent, in (1), not only upon the choice of a possible future, but also upon the choice, triggered by the indefinite temporal adverb, of a time in that future:

($1_{st}$)  (1) is (super)true at $\alpha$ iff  $(\forall h \in H_\alpha)(\exists \beta \in h)[\alpha < \beta$ *and* $(\forall p_{B,\beta})$['The number of trees in Brisbane is even' is (bivalently) true at the complete model $\beta(p_{B,\beta})$]

The two universal quantifiers in ($1_{st}$) mark the two occurences of the Chance move in $\Gamma[(1)]$, and the fact that an occurrence of existential quantifier found its way between the two seems to make the problem of equivalently transforming $\Gamma[(1)]$ into a chance-prenexed game seems insurmountable. Just there seems to be no way of transforming, in ($1_{st}$), the $\forall\exists\forall$-prefix into a $\forall\forall\exists$-prefix.

Thus, $\Gamma[(1)]$ cannot be reduced straightforwardly, if at all, to a chance-prenexed semantic game. One of the two sources of supervaluations involved in the semantics of (1), under the preferred reading, seems to be inherently *nested* in the depths of the associated semantic game.

This implies, in particular, that the rigid position and the reference-point approaches are hopeless, as regards sentence (1) and its like. The semantics of such sentences forces us to alternate supervaluational moves that partially resolve the incompleteness of the initial model with moves of a different nature that involve the Verifier's choices.

## 5. Summing up

On the logician's side, the most deplorable implication of the fact that natural languages like English harbour sentences that appeal to nested supervaluations is that the classicality of the issuing logic gets endangered.

Let S be (1), and consider $S \vee \neg S$:

(2)  Either it is the case that sometimes in the future, the number of trees in Brisbane will be even or it is not the case that sometimes

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in the future, the number of trees in Brisbane will be even.

Given the semantic game $\Gamma[(1)]$, how should we construct $\Gamma[(2)]$? Presumably, we should stick by the Uncork-Re-Cork Construction Rule (cf. Section 2 above) whose very point has been to secure the preservation of classicality.

But the trouble is that in this case we are just not in a position to follow the Uncork-Re-Cork Rule in its entirety. For suppose that we have (i) removed the initial chance moves from both $\Gamma(S; \alpha)$ and $\Gamma(\neg S; \alpha)$; (ii) for any history $h \in H_\alpha$, constructed the subgame $\Gamma(S \vee \neg S; \alpha, h)$ on the basis of $\Gamma(S; \alpha, h)$ and $\Gamma(\neg S; \alpha, h)$; (iii) re-attached the initial chance move to the family of newly constructed subgames of the form $\Gamma(S \vee \neg S; \alpha, h)$. Still, we have *not* removed (and how could we?) the nested supervaluational moves in the process; the subgames $\Gamma(S; \alpha, h)$ and $\Gamma(\neg S; \alpha, h)$ are *not* bivalent; they each contain a chance move, and a non-initial one, at that. And this may become fatal for the (super)truth of $S \vee \neg S$ at $\alpha$. In fact, it is straightforward to see that in a situation in which neither the Verifier nor the Falsifier has a winning strategy in $\Gamma(S; \alpha, h)^6$, neither of them has a winning strategy in $\Gamma(\neg S; \alpha, h)$. Consequently, $S \vee \neg S$ is not (super)true at $\alpha$.

On the side of linguistical semantics, though, the news that some natural-language sentences feature nested supervaluations need not be a bad one, if the natural-language semanticist's main task is to faithfully characterise whatever semantic phenomena may be "out there" in the language under scrutiny. If my observations and argumentation are correct, there is a semantic framework that proved to be adequate in treating (or at least in describing) nested supervaluations in English, namely GTS with its lexicalist approach toward constructing the structured semantic representations of English sentences.

## REFERENCES

1. *Blinov A*. Semantic Games with Chance Moves // Synthese. 1994. Vol. 99. P. 311-327.
2. *Fraassen B.C. van*, Singular Terms, Truth-Value Gaps and Free Logic // Journal of Philosophy. 1966. Vol. 63. P. 481-495.
3. *Fraassen B.C. van*. Presupposition, Implication and Self-reference // Journal of Philosophy. 1968. Vol. 65. P. 136-152.
4. *Fraassen B.C. van*. Formal Semantics and L*ogic //* London: Macmillan, 1971.
5. *Hintikka J., Kulas J*. The Game of Language Studies // Game-Theoretical Semantics and its Applications. Dordrecht: D. Reidel, 1983.

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[6] This is the case when at every future time ⧠⧠ *h* 'The number of trees in Brisbane is even' is a gap.

6. *Hintikka J*., *Sandu G.* On the Methodology of Linguistics: A Case Study // Oxford: Blackwell, 1991.
7. *Hintikka J., Sandu G.* Game-Theoretical Semantics // J. van Benthem and A. ter Meulen (eds.), Handbook of Logic and Language. Amsterdam: Elsevier, and Cambridge, MA: MIT Press, 1997.
8. *Hodges W.* Elementary Predicate Logic // D.Gabbay and F.Guenthner (eds.), Handbook of Philosophical Logic. Dordrecht: D. Reidel, 1983. Vol. 1 P.1-131.
9. *Saarinen E.* (ed.). Game-Theoretical Semantics // Dordrecht: D. Reidel, 1979.
10. *Thomason R.H.* Indeterminist Time and Truth-Value Gaps // Theoria. 1970. Vol. 36. P. 264-281.