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***Неклассическая логика***

*Non-classical Logic*

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# Model checking for coalition announcement logic

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**Abstract:** This talk is based on joint work with Rustam Galimullin and Hans van Ditmarsh, published in the German Conference on Artificial Intelligence (KI 2018). First I will introduce background and motivation for the work. I will introduce multi-agent Epistemic Logic (EL) for representing knowledge of (idealised) agents, Public Announcement Logic (PAL) for modelling knowledge change after truthful announcements, Group Announcement Logic (GAL) for modelling what kinds of changes in other agents' knowledge a group of agents can effect, and Coalition Announcement Logic (CAL) which is the main subject of the talk. CAL studies how a group of agents can enforce a certain outcome by making a joint announcement, regardless of any announcements made simultaneously by the opponents. The logic is useful to model imperfect information games with simultaneous moves. It is also useful for devising protocols of announcements that will increase some knowledge of some agents, but also preserve other agents' ignorance with respect to some information (in other words, preserve privacy of the announcers). The main new technical result in the talk is a model checking algorithm for CAL, that is, an algorithm for evaluating a CAL formula in a given finite model. The model-checking problem for CAL is PSPACE-complete, and the protocol requires polynomial space (but exponential time).

**Keywords:** epistemic logic, public announcement logic, coalition announcement logic, model checking algorithm

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## 1. What this talk is about

Logics for describing announcements by (groups of) agents, and how announcements affect agents' knowledge. More precisely, model checking algorithm for Coalition Announcement Logic.

The report based on the paper just accepted for the German Conference on Artificial Intelligence (KI 2018).

First, we will introduce some background on logic of knowledge, logic of public announcements, and logics of group and coalition announcements.

### 1.1. Multi-agent epistemic logic: example

For more detailed exposition of epistemic logic see [Hintikka, 1962].

Consider the following example: there are three agents, $a$, $b$ and $c$.

Suppose that, $a$ and $b$ are households that either consume or not consume power ($p_1$ is true if $a$'s power is on, and $p_2$ is true if $b$'s power is on). $c$ is an electricity substation that needs to know how many households consume power, but not whether individual households consume power or not.

We describe above the situation of $c$ not knowing anything about power consumption (and $a$ and $b$ knowing their own and each other's status).

Possible worlds (or states) are $w_0$, $w_1$, $w_2$, $w_3$ with different truth values of $p_1$ and $p_2$. In $w_0$, $a$ and $b$ know that they are in $w_0$ and $\neg p_1 \wedge p_2$ is true $c$ does not know whether $p_1$ and $p_2$ are true (see Fig. 1):

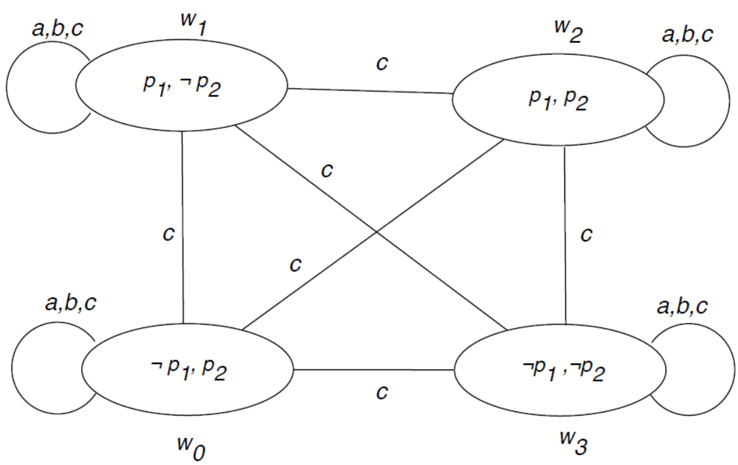


Fig. 1. Example 1.

## 2. Multi-agent epistemic logic

Fix a non-empty finite set of agents $A$ and a set of propositional variables $P$.

**Definition 1.** Kripke model A *Kripke model* is a triple $M = (W, \sim, V)$, where

- $W$ is a non-empty set of states,
- $\sim: A \to \mathcal{P}(W \times W)$ assigns an equivalence relation to each agent, and

- $V : P \to \mathcal{P}(W)$ assigns a set of states to each propositional variable.
- $M$ is called *finite*, if $W$ is finite.
- A pair $(M, w)$ with $w \in W$ is called a *pointed model*, where $w \in W$ is an actual world.

Consider the following *epistemic language* $\mathcal{L}_{EL}$:

$$\varphi, \psi ::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid K_a\varphi$$

where $p \in P$, $a \in A$, and all the usual abbreviations of propositional logic and conventions for omitting parentheses hold. $K_a\varphi$ stands for '$a$ knows that $\varphi$'. The dual operator '$a$ considers $\varphi$ possible' $\widehat{K}_a\varphi$ is defined as $\neg K_a \neg\varphi$.

Forcing relation is defined as follows:

$$\begin{array}{lll} (M,w) \models p & \text{iff} & w \in V(p) \\ (M,w) \models \neg\varphi & \text{iff} & (M,w) \not\models \varphi \\ (M,w) \models \varphi \wedge \psi & \text{iff} & (M,w) \models \varphi \text{ and } (M,w) \models \psi \\ (M,w) \models K_a\varphi & \text{iff} & \forall v \in W : w \sim_a v \text{ implies } (M,v) \models \varphi \end{array}$$

Let us consider the following example:

$$\begin{array}{l} (M, w_0) \models K_a(\neg p_1 \wedge p_2) \\ (M, w_0) \models \neg K_c(\neg p_1 \wedge p_2) \\ (M, w_0) \models \neg K_c \neg p_1 \end{array}$$

See the Example 1 on Fig. 1.

## 3. Public Announcement Logic (PAL)

Public announcement logic was initally proposed by Plaza [Plaza, 2007].

Let us consider the following examples:

Suppose $c$ hears that $(\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)$ is true. Some worlds become impossible (see Fig. 2).

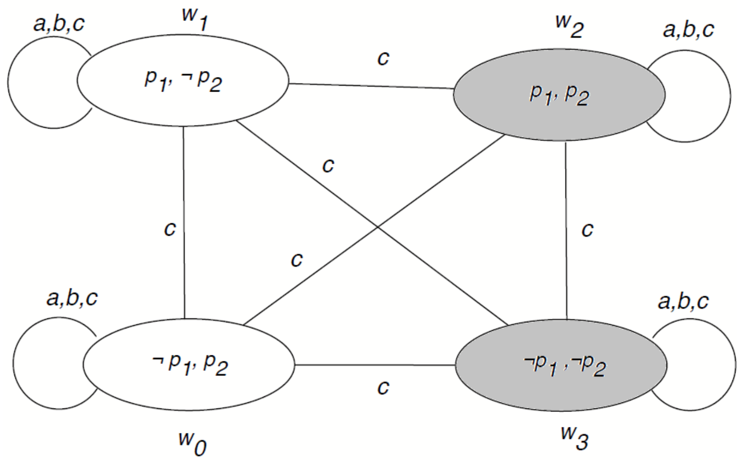


Fig. 2. Example 2.

Suppose $c$ hears that exactly one of the households consumes power: $(\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)$. After this announcement, $c$ only considers $w_0$ and $w_1$ possible (see Fig. 3).

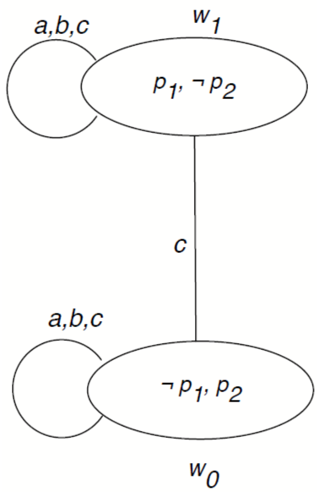


Fig. 3. Example 3.

### 3.1. Definitions

The language of PAL is $\mathcal{L}_{PAL}$:

$$\varphi, \psi ::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid K_a\varphi \mid [\psi]\varphi,$$

where as before, $p \in P$, $a \in A$.

$[\psi]\varphi$ stands for 'after $\psi$ is truthfully announced, $\varphi$ holds'.

The dual operator $\langle\psi\rangle\varphi$ is defined as $\neg[\psi]\neg\varphi$ and means '$\psi$ is true, and after it is announced, $\varphi$ is true'.

Given $(M, w)$ and $\varphi \in \mathcal{L}_{EL}$, an *updated model* $(M, w)^\varphi$ is a restriction of the original model to the states where $\varphi$ holds.

$$\begin{array}{lll} (M, w) \models [\varphi]\psi & \text{iff} & (M, w) \models \varphi \text{ implies } (M, w)^\varphi \models \psi \\ (M, w) \models \langle\varphi\rangle\psi & \text{iff} & (M, w) \models \varphi \text{ and } (M, w)^\varphi \models \psi \end{array}$$

## 4. Group and Coalition Announcement Logics (GAL and CAL)

Intuition: announcements by (group of) agents:

- announcements are made by agents
- agents can only announce what they know
- for example, $a$ can announce $K_a((\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2))$, but $c$ can not
- the only thing $c$ can announce in $(M, w_0)$ is $K_c\top$
- a group of agents can announce a conjunction of formulas, each formula known by an agent in the group.

The language $\mathcal{L}_{GAL}$ of group announcement logic [Ågotnes, van Ditmarsch, 2008] is $\mathcal{L}_{PAL}$ extended with $\langle G\rangle\varphi$, where $G \subseteq A$, which stands for 'there is a truthful announcement by $G$, after which $\varphi$ holds'. Let $\mathcal{L}^G_{EL}$ denote the set of formulas of the type $\bigwedge_{a \in G} K_a\varphi_a$, where for every $a \in G$ it holds that $\varphi_a \in \mathcal{L}_{EL}$.

$$(M, w) \models \langle G\rangle\varphi \text{ iff } \exists\psi{\in}\mathcal{L}^G_{EL} : (M, w) \models \langle\psi\rangle\varphi$$

The language $\mathcal{L}_{CAL}$ [Ågotnes et all, 2010] of coalition announcement logic is $\mathcal{L}_{PAL}$ extended with $\langle\!\lbrack G\rbrack\!\rangle\varphi$, where $G \subseteq A$, which stands for 'there is an announcement by $G$ such that whatever agents in $A \setminus G$ announce simultaneously, afterwards $\varphi$ holds':

$$(M, w) \models \langle\!\lbrack G\rbrack\!\rangle\varphi \text{ iff } \exists\psi{\in}\mathcal{L}^G_{EL}\ \forall\chi{\in}\mathcal{L}^{A\setminus G}_{EL} : (M, w) \models \psi \wedge [\psi \wedge \chi]\varphi$$

- $a$ and $b$ together can make an announcement after which $K_c(((\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)) \wedge \neg K_c \neg p_1)$ holds: $(M, w_0) \models \langle a, b\rangle K_c(((\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)) \wedge \neg K_c \neg p_1)$; also,
  $(M, w_0) \models \langle\!\langle a, b\rangle\!\rangle K_c(((\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)) \wedge \neg K_c \neg p_1)$
- $a$ can make an announcement after which $K_c(((\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)) \wedge \neg K_c \neg p_1)$ holds: $(M, w_0) \models \langle a\rangle K_c(((\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)) \wedge \neg K_c \neg p_1)$
- $a$ cannot make an announcement after which $K_c(((\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)) \wedge \neg K_c \neg p_1)$ holds, *no matter what other agents announce simultaneously* (because $b$ can announce $K_b p_2$):

$$(M, w_0) \not\models \langle\!\langle a\rangle\!\rangle K_c(((\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)) \wedge \neg K_c \neg p_1)$$

## 5. Model checking CAL

Model checking problem for a logic $L$: given a (finite) model $M$ of $L$ and a formula $\phi$ of $L$, does it hold that $M \models \phi$?

Model-checking problem for CAL: given a pointed Kripke model $(M, w)$ and a formula $\phi$ of CAL, does it hold that $M \models \phi$?

The model checking problem for CAL is interesting because we can use it to plan epistemic actions. What can we tell other agents so that we are guaranteed to get just the right information to them without revealing too much. For example, epistemic planning and verification of distributed protocols.

**Definition 2.** Bisimulation Let two models $M = (W, \sim V)$ and $M' = (W', \sim',$ $V')$ be given. A non-empty binary relation $Z \subseteq W \times W'$ is called a bisimulation if and only if for all $w \in W$ and $w' \in W'$ with $(w, w') \in Z$:

- $w$ and $w'$ satisfy the same propositional variables;
- for all $a \in A$ and all $v \in W$: if $w \sim_a v$, then there is a $v'$ such that $w' \sim_a v'$ and $(v, v') \in Z$;
- for all $a \in A$ and all $v' \in W'$: if $w' \sim_a v'$, then there is a $v$ such that $w \sim_a v$ and $(v, v') \in Z$.

Let $(M, w), (M^{'}, w^{'})$ are pointed models and $Z \subseteq W \times W^{'}$ is a bisimulation, then $(M, w)$ and $(M^{'}, w^{'})$ are bisimilar.

If $Z_1, Z_2$ are bisimulations, then $Z_1 \cup Z_2$ is a bisimulation. Union of all bisimulations is a maximal bisimulation.

**Definition 3.** The *quotient model* of $M$ with respect to some relation $R$ is $M^R = (W^R, \sim^R, V^R)$, where $W^R = \{[w] \mid w \in W\}$ and $[w] = \{v \mid wRv\}$, $[w] \sim_a^R [v]$ iff $\exists w' \in [w]$, $\exists v' \in [v]$ such that $w' \sim_a v'$ in $M$, and $[w] \in V^R(p)$ iff $\forall w' \in [w] : w' \in V(p)$.

**Definition 4.** *Bisimulation contraction* of $M$ (written $|M|$) is the quotient model of $M$ with respect to the maximal bisimulation of $M$ with itself, i.e. bisimulation contraction is the minimal representation of M.

**Definition 5.** A model $M$ is *bisimulation contracted* if $M$ is isomorphic to $|M|$.

**Lemma 1.** *$(|M|, w) \models \varphi$ iff $(M, w) \models \varphi$ for all $\varphi \in \mathcal{L}_{CAL}$.*

Every pointed model $(M, w)$ is distinguished from all other non-bisimilar pointed models $(M, v)$ by some distinguishing formula $\delta_w \in \mathcal{L}_{EL}$.

Given a finite model $(M, w)$, distinguishing formula $\delta_w$ is constructed recursively as follows:

$$\delta_w^{k+1} ::= \delta_w^0 \land \bigwedge_{a \in A} ( \bigwedge_{w \sim_a v} \widehat{K}_a \delta_v^k \land K_a \bigvee_{w \sim_a v} \delta_v^k),$$

where $0 \leq k < |W|$, and $\delta_w^0$ is the conjunction of all literals that are true in $w$, i.e. $\delta_w^0 ::= \bigwedge_{w \in V(p)} p \land \bigwedge_{w \notin V(p)} \neg p$.

A *distinguishing formula for a set of states $S$* is

$$\delta_S ::= \bigvee_{w \in S} \delta_w.$$

See Fig. 4, 5, 6, 7.

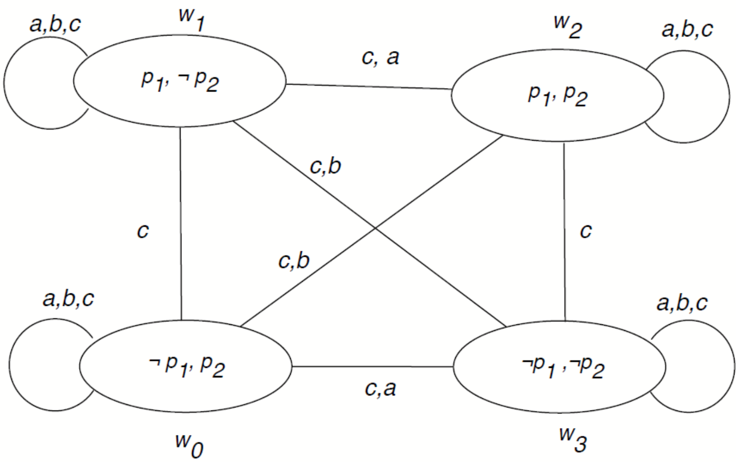


Fig. 4. Example 4 (Alternative model $M'$: $a$ and $b$ only know “their” variable).

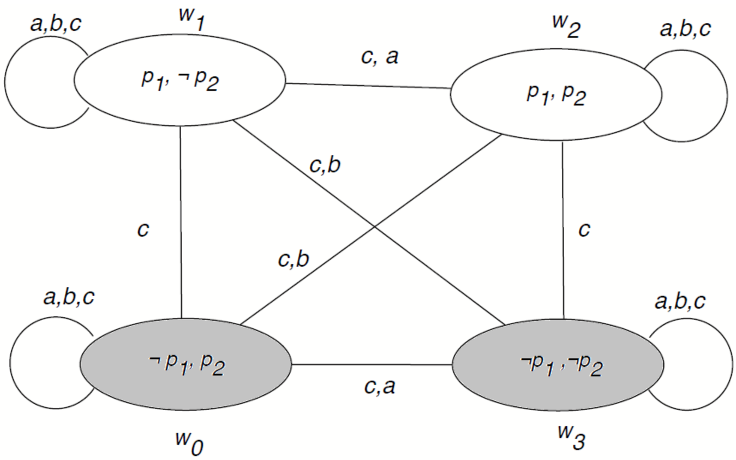


Fig. 5. Example 5 ($a$'s equivalence in $w_0$: can announce $K_a \neg p_1$).

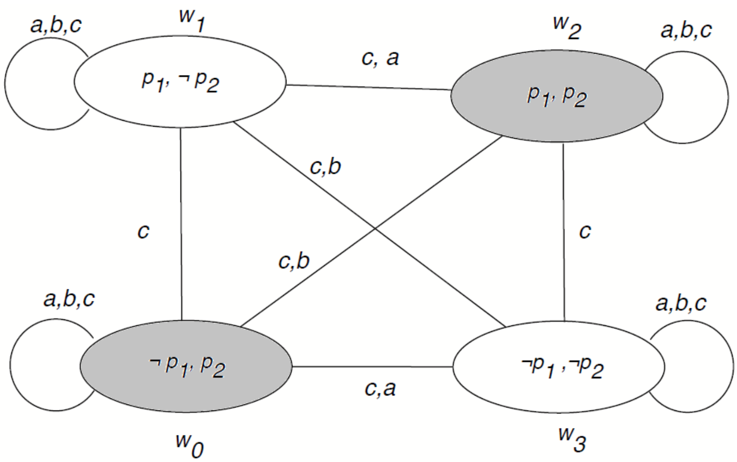


Fig. 6. Example 6 ($b$'s equivalence in $w_0$: can announce $K_b p_2$).

Let $M/a = \{[w_1]_a, \dots, [w_n]_a\}$ be the set of $a$-equivalence classes in $M$.

**Definition 6.** A *strategy* $X_a$ for an agent $a$ in a finite model $(M, w)$ is a union of equivalence classes of $a$ including $[w]_a$.

**Definition 7.** *The set of all available strategies* of $a$ is $S(a, w) = \{[w]_a \cup X_a : X_a \subseteq \bigcup M/a\}$.

**Definition 8.** *Group strategy* $X_G$ is defined as $\cap_{a \in G} X_a$ for all $a \in G$. The set of available strategies for a group of agents $G$ is $S(G, w) = \{\cap_{a \in G} X_a : X_a \in S(a, w)\}$.

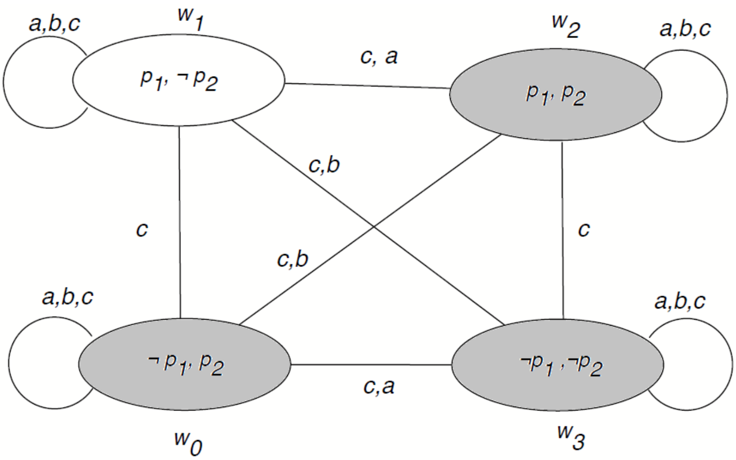


Fig. 7. Example 7 ($w_0$ is the intersection of $a$'s and $b$'s equivalence classes in $w_0$: can announce $K_a \neg p_1$ and $K_b p_2$).

Given a finite and bisimulation contracted model $(M, w)$ and strategy $X_G$, a distinguishing formula $\delta_{X_G}$ for $X_G$ is $\bigvee_{w \in X_G} \delta_w$.

In a bisimulation contracted model we propose alternative truth definition for $\langle\!\langle G \rangle\!\rangle$,

$$(M, w) \models \langle\!\langle G \rangle\!\rangle \varphi \text{ iff } \exists X_G \in S(G, w)\, \forall X_{A \backslash G} \in S(A \backslash G, w) : (M, w)^{X_G \cap X_{A \backslash G}} \models \varphi.$$

## 6. Model checking algorithm

Algorithm $mc(M, w, \varphi_0)$, where $(M, w)$ is a pointed model and $\varphi_0$ is some formula:

**case** $\varphi_0$

$p$ : **if** $w \in V(p)$ **then return** $true$ **else return** $false$;

$\neg\varphi$ : **if** $\neg mc(M, w, \varphi)$ **then return** $true$ **else return** $false$;

$\varphi \wedge \psi$ : **if** $mc(M, w, \varphi)$ and $mc(M, w, \psi)$ **then return** $true$ **else return** $false$;

$K_a\varphi$ : **for all** $v \sim_a w$
  **if** $\neg mc(M, v, \varphi)$ **then return** $false$;
  **return** $true$

$\langle\psi\rangle\varphi$ **if** $\neg mc(M, w, \psi)$ **then return** $false$, **else** compute the $\psi$-submodel of $M$ and **return** $mc(M^\psi, w, \varphi)$.

$\langle\!\langle G\rangle\!\rangle\varphi$**:** compute $(|M|, w)$ and sets of strategies $S(G, w)$ and $S(A \setminus G, w)$
**for all** $X_G \in S(G, w)$
. *check* = *true*;
. **for all** $X_{A\setminus G} \in S(A \setminus G, w)$
. **if** $\neg mc(|M|^{X_G \cap X_{A\setminus G}}, w, \varphi)$ **then** $check = false$
. **if** *check* **then return** *true*
**return** *false*.

**Theorem 1.** *The model checking problem for CAL is PSPACE-complete.*

## 7. Summary

We can use model checking to verify properties of announcements (for example, communication protocols, or data collection).

We can also use it to produce strategies (the right announcements to make) given the properties that should hold after the announcement

## 8. Questions

Question (V.I. Shalack): All these epistemic logic are S5, so we have got a problem with paradox of omniscience. Is there some another way to interpret knolwedge differently? For example, in jurisprudence and legal practice we are led by the principle that ignorance of law was no excuse, i.e. if a law was accepted, than everyone should know this laws and its consequences.

Answer: I prefer the syntactical interpretation of an epistemic modality, where every agent has his own 'knowledge set' of formulas, with their own rules how to infer consequences from these formulas. This way allows to model epistemic situations without logical omniscience. I agree that S5 is not always a good logic for modelling knowledge. On the other hand, if we work with a small number of formulas, then it is less paradoxical to assume omniscience limited to these formulas only.

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