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ON THE COGNITION OF LAWS OF NATURE

In this paper I shall discuss the problem of cognition of laws of nature on the following different levels of understanding:

(i) First level of understanding of laws of nature: the Greek Ideal of Science
(ii) Second level: Space Time Invariance
(iii) Third level: Dynamical Laws
(iv) Fourth level: Statistical Laws
(v) Fifth level: Laws and Causality
(vi) Sixth level: Chaotic Motion
(vii) Seventh level: Initial conditions and Constants of Nature

Before I shall begin with the first level of cognition or understanding a short clarification of different meanings of the expression 'law of nature' will be given:

L1 the "law" as it "is" in the thought of the inventor or discoverer

L2 The "law" as it "is" in the things which are ordered or described by it

L3 The "law" as a law statement formulated in some scientific language

L4 The "law" as an ideal true law, w.r.t. which laws known at present in the sense of L3 are approximations

L5 The "law" as ideal conceptual entity more or less independent and separated from law statements.

L3 is preferable to L1, since what an author like Newton thought (for example about his law of gravitation) is open to speculations, even if historians of science may find out something about it; but then also from written documents. The existence of L2 expresses a modest realism: What corresponds to a true law of nature is a structure of things (natural objects) with their properties and relations among them (without assuming a naive picture theory). Observe however that there are important differences between L2 and L3: Law statements are true or false, can be tested, confirmed, refuted, they can be nearer to the truth than other law statements, they can contain negations… etc. Nothing of these features can hold for structures of real objects. Concerning L4 one may say that a law statement L3 is usually an approximation to the true law L4 in Popper's sense: a law L is a better approximation to L4 than another law L’ iff L has more true relevant and informative consequences and less false ones than L’.[1] Sense L5 of law comes close to Bolzano’s and Frege’s view about laws of logic and mathematics and Popper’s interpretation of them by abstracting from the knowing subject and with the help of his theory of the 3rd world. It seems, however, that L5 is more suitable for laws of logic and mathematics than for laws of nature. In conclusion I want to say that the expression ‘law of nature’ is used in this essay – if not otherwise indicated – in the sense of L3.

Laws in the sense of L3 can still be used in different ways: as laws about natural objects or systems (for example physical systems) and as (meta)laws about laws. Especially the second type can be used in a descriptive or normative way. Thus the principle of Special Relativity can be expressed first as a law about physical systems or physical reference frames: all inertial systems are equivalent; secondly as a metalaw about laws: all laws of nature are invariant under changes (transformations) of inertial reference frames; thirdly as a methodological norm about laws: all laws of nature should be invariant under changes of inertial reference frames.

First Level: the Greek Ideal of Science

In order to be able to describe and explain movement we need to distinguish something which changes relative to something which does not change. This important distinction is pointed out by Aristotle[2] also as a criticism of Parmenides’ theory of the universe which assumes only one being and nothing else.[3] That what changes, moves was thought to be contingent (not necessary) in respect to the not changing (or even not changeable) necessary principle or law. In general this idea belongs to the Greek Ideal of Science which was more or less manifest in several Greek thinkers from Thales on but was elaborated in detail by Plato and Aristotle:

To describe and explain the visible (observable), concrete, particular, changing, material contingent world by non-visible (non-observable) abstract, universal, non changing immaterial and necessary principles.

This first level of cognition of laws of nature which is manifest in the Greek Ideal of Science teaches us that our understanding of any

[1] That this concept of approximation to truth (Verisimilitude) can be made precise (and freed from the objections by Tychy and Miller) was shown in Schurz-Weingartner (1987) and Weingartner (2000) ch. 9.

[2] Aristotle (Phys) 190a17f.

[3] Aristotle (Met) 986b15f. and (Phys) 186a24f.

kind of genuine law is such that a law is something which does not change, i.e. is invariant (symmetric) relative to something else which changes:

"In fact it may be argued that laws of nature could not have been recognized if they did not satisfy some elementary invariance principles – such as translation in Euclidean space and translation in time – if they changed from place to place, or if they were also different at different times"[4].

"Physicists as a rule hold that physical laws are eternal… It is indeed difficult to think otherwise, since what we call the laws of physics are the results of our search for invariants. Thus even if a supposed law of physics should turn out to be variable, so that (say) one of the apparently fundamental physical constants should turn out to change in time, we should try to replace it by a new invariant law that specifies the rate of change"[5].

As it will be clear from the above considerations and from the two quotations a first level of understanding of a genuine law is that it expresses an invariance.

Second Level: Space Time Invariance

Whereas the first level of understanding a law is concerned with invariance in general the second level of understanding is concerned with finding out a specific kind of invariance. Among these the oldest and most famous one is the invariance w.r.t. space and time. Or in other words: the invariance under changes of place and time.

"The paradigm for symmetries of nature is of course the group of symmetries of space and time. These are symmetries that tell you that the laws of nature don't care about how you orient your laboratory, or where you locate your laboratory, or how you set your clocks or how fast your laboratory is moving"[6].

In this quotation Weinberg describes roughly the four important kinds of space time invariance or invariance w.r.t. continuous change of space time. These four kinds can be described in more detail as follows: The first three are not concerned with real (permanent) movement but only with changing place, orientation and delay of time. The last is more complicated and we have to split it also into three different types.

(i) Location of Laboratory: Laws are invariant w.r.t. the place of the laboratory. This is called translation symmetry (invariance) in space. It yields three conservation laws of momentum. And since

[4] Wigner (1967) p. 43
[5] Popper-Eccles (1981) p. 14
[6] Weinberg (1987) p. 73

every new invariance (symmetry) implies a new unobservable, what becomes unobservable here is absolute place. That means that the laws of nature do not designate a certain place. They abstract from *hic* (here) and *nunc* (now) as Thomas Aquinas pointed out already[7].

(ii) Orientation of laboratory: Laws are invariant w.r.t. the orientation of the laboratory. This is called rotation symmetry in space. Observe however that the laboratory is not rotating it is just reorientated (turned on an angle). This yields three conservation laws of angular momentum. Unobservable: absolute direction.

(iii) Clock setting in laboratory: Laws are invariant w.r.t. to time delay; i.e. it does not matter how you set your clocks in the laboratory. This yields the conservation law of energy. Unobservable: absolute point of time. Concerning the movement of the laboratory (you may always insert 'reference frame' instead of 'laboratory') three different kinds have to be distinguished:

(iv) Inertial movement I: Inertial movement is force free movement of particles (mass points) with uniform velocity along straight lines. For inertial movement I there are three further assumptions: (i) a uniform time scale (or equal time measurement) everywhere (ii) Euclidean Space (iii) with velocity much smaller than the velocity of light. The invariance defined by (1)-(4) is called Galilean Invariance. All laws of Classical Mechanics are Galilei invariant (or: invariant under Galilei transformations). But Maxwell's laws are not.

(v) Inertial movement II: for inertial movement II there is only the assumption that $v = c$, i.e. that the velocity v is smaller than or equal to the light velocity c. Observe however that conditions (i) and (ii) are dropped. That means that (i) it is permitted that the time measurement may be different w.r.t. to inertial reference frames (laboratories) which move with different velocity; and (ii) that Euclidean space is replaced by Minkowski space. The invariance defined by (1)-(3) and (5) is called Lorentz Invariance or invariance of Special Relativity. Maxwell's laws are Lorentz invariant. The laws of Classical Mechanics are not, but they can be revised by Einstein's (and Lorentz's) corrections in order to become Lorentz invariant. The great idea of Einstein (1905) w.r.t. understanding of what genuine laws of nature are was this: One can keep all laws of Classical Mechanics and of Maxwell's Theory invariant under inertial transformations II (i.e. without taking into account gravitation or strong forces) by giving up assumptions (i)-(iii) of (4) and assuming the constancy of light velocity (that the propagation of

[7] Cf. (STh) I, 46,2.

light with velocity c is independent of the movement of the source). A consequence is then that the magnitudes m, l, and t are not invariant under movement II, - i.e. if v comes close to c – and have to be corrected by the factor defined in the Lorentz transformation. This idea was however not just a mathematical advice for reformulation, because the consequences (the respective changes of the magnitudes: growing mass, length contraction, time dilatation) could be confirmed by experiment.

(vi) Arbitrary movement or arbitrary space time transformations. This kind of invariance of laws under arbitrary space time transformations is the invariance of general Relativity. It is the most general invariance of laws of nature we know today. In order to understand it more accurately, we have to notice that it transcends Lorentz Invariance in a threefold way: (i) first in the sense that it drops the restriction to inertial reference frames, (ii) secondly in the sense that it drops the restriction to straight Galilean coordinates, and (iii) third in the sense that it extends to gravitation.

Laws of nature which are invariant in that sense are Einstein's field equations.

Third Level: Dynamical Laws

From the time of Newton on one had a better and better understanding of one important type of law: the dynamical law. This understanding was connected with conditions satisfied by those physical systems the behaviour of which was successfully described by these dynamical laws; i.e. in such a way that the laws were confirmed by the application:

D1 the state of the physical system S at any given time t_i is a definite function of its state at an earlier time t_{i-1}. A unique earlier state (corresponding to a unique solution of the differential equation) leads under the time evolution to a unique final state (again corresponding to an unique solution of the equation).

D2 condition D1 is also satisfied for every part of the physical system, especially for every individual body (object) as part of the system even if the individual objects may differ in the classical or in the quantum mechanical sense.

D3 the physical system S is periodic, that is the state of S repeats itself after a finite period of time and continues to do so in the absence of external disturbing forces.

D4 the physical system S has a certain type of stability which obeys the following condition: Very small changes in the initial states, say within a neighbourhood distance of ε lead to proportionally small (no more than in accordance of linearly increasing function of time) changes *h(ε)* in the final state. This kind of stability which survives small perturbations and leads to relaxation afterwards is called *perturbative stability* and holds in many linear systems[8].

D1 is the main condition for dynamical laws although D3 and D4 were underestimated and D4 was only understood more deeply after the discovery of chaotic motion (see Sixth Level). D1 is manifest in Laplace's view that the dynamical laws are *the* laws governing our universe: "We ought to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow"[9]. In this sense Laplace thought that all the states of the universe could be calculated i.e. predicted and retrodicted from one single state with the help of the (dynamical) laws. Along this line of reasoning D1 is usually taken as the defining condition for determinism; where determinism is thought to imply predictability and conservation of information. But this is not correct because D1 does not guarantee predictability or conservation of information if D4 is not satisfied (see below).

D2 states that the dynamical law describes also the time development of the individual particle or object as a part of the time development of the whole physical system. This presupposes a concept of 'individual object' which is re-identifiable through time as understood in Classical Mechanics. Such kinds of objects are not available in this straightforward way (but only with special constraints) in Quantum Mechanics.

D3 is not a necessary condition for the application of dynamical laws obeying D1 and D2 though D3 is satisfied in most cases where dynamical laws are applied. The main point is that according to D3 there is recurrence of the state of the physical system after some finite period of time.

Are there important cases of physical system which satisfy D1 but not D3? The answer to this question is Yes. The systems in question are systems which show chaotic behaviour (or systems in chaotic motion). Chaotic behaviour is non-periodic. And this holds also without any external disturbance. A consequence of that is a further characteristic of chaotic motion: The Poincaрй map shows space-filling points. This is a method introduced by Poincaрй about 100 years ago which considers the points in which the trajectory cuts a certain plane. If the motion is chaotic there will be no immediate recurrence that is the plane will always be cut at new points and as time goes on will be filled with

[8] Cf. the discussion of the conditions D1,D3 and D4 in Holt-Holt (1993).
[9] Laplace (1814) ch. 2

points. But if the phase space is small – chaotic motion is bounded motion – there will be recurrance of the trajectory after some finite period of time.

D4 was a hidden assumption of Classical Mechanics (CM) until the end of the 20th century. In other words the laws of CM were understood in such a way that D4 is always satisfied. The neglect is expressed by Lighthill as follows:

"Here I have to pause, and to speak once again on behalf of the broad global fraternity of practitioners of mechanics. We are all deeply conscious today that the enthusiasm of our forebears for the marvellous achievements of Newtonian mechanics led them to make generalizations in this area of predictability which, indeed, we may have generally tended to believe before 1960, but which we now recognize were false. We collectively wish to apologize for having misled the general educated public by spreading ideas about the determinism of systems satisfying Newton's laws of motion that, after 1960 were to be proved incorrect"[10]. On the other hand, that there are cases which violate D4, i.e. cases where small initial deviations lead to un-proportional (expo-nentionally increasing) effects was known by experienced people from antiquity (see below, level 6: chaotic motion).

Fourth Level: Statistical Laws

After the discovery of statistical laws in thermodynamics and later in other areas there was a general doubt with respect to the mechanistic and deterministic interpretation of the world with the help of dynamical laws. That there are physical truths which are statistical in character was clear for Boltzmann and Poincarй who both underline the importance of Maxwell's, Clausius', Gibbs' and Carnot's discoveries[11].

The question was now: Could it not be the case that all laws are statistical and the deterministic outlook is only on the surface of macroscopic phenomena? That is all complex systems of the world are in fact, in its inmost structure, i.e. on the atomic level, like gases or swarms of mosquitos or clouds. And how can a law then emerge from such a random behaviour of milliards of gas molecules? Schrцdinger gave the following answer in his inaugural lecture in Zьrich (1922):

"In a very large number of cases of totally different types, we have now succeeded in explaining the observed regularity as completely due to the tremendously large number of molecular processes that are co-operating. The individual process may, or may not, have its own strict regularity. In the observed regularity of the mass phenomenon the indi-

[10] Lighthill (1986) p. 38.
[11] Cf. Boltzmann (1896) p 567 and Poincarй (1958) p. 97

vidual regularity (if any) need not be considered as a factor. On the contrary, it is completely effaced by averaging millions of single processes, the average values being the only things that are observable to us. The average values manifest their own purely statistical regularity…"[12].

The main points in which statistical laws differ from dynamical laws can be expressed by the following four conditions S1-S4 which refer to D1-D4 respectively:

S1 the state of the physical system at t_i is not a definite function of an earlier state at t_{i-1} . The same initial state may lead to different successor states (branching);

S2 statistical laws describe and predict the states of the whole physical system but they do not describe or predict the individual parts (objects) of this system;

S3 statistical laws describe only physical systems which are non-periodic, i.e. systems with extremely un-probable recurrence of the whole state of the system;

S4 the loss of information (and consequently the difficulty of prediction) about the state of an individual object (or a small part) of the whole system increases exponentially with the complexity of the system. On the other hand: the (accuracy of the) information about the average values of magnitudes (parameters) of the state of an individual object (or small part) increases also with the complexity of the system.

There were two main questions concerning both types of laws:

(vii) Is one type of law reducible to the other?

(viii) Are statistical laws compatible with dynamical laws?

4.1 Statistical Laws are not reducible to dynamical laws

The answer to the first question is certainly: No. This can be shown from a comparison of the conditions D1-D4 to S1-S4.

a. It is easy to see that there is an essential difference between the conditions D1 and S1. Like D1 is necessary for dynamical laws, S1 is necessary for statistical laws. This presupposes however that we interpret S1 (and by it statistical laws) realistically. That is we assume there is real branching in reality. An epistemic (or idealistic) interpretation according to which branching is only a sign for our lack of knowledge whereas in the underlying reality everything is determined (by hidden parameters and dynamical laws of which we are ignorant) we do not find justified. This can be substantiated

[12] Schrudinger (1961) p. 11. Schrudinger's lecture which had the title „Was ist ein Naturgesetz" was later published in the book "Was ist ein Naturgesetz?" which is a collection of essays by Schrudinger.

by the fact that the following types of processes do not satisfy D1 (but satisfy S1) as is evident by all the sophisticated knowledge we possess today about these processes: Thermodynamical processes, processes of friction, of diffusion, of radiation, of electric transport, processes of Quantum Mechanics, processes of biology, of cosmology and of psychology[13].

b. Similarly D2 and S2 differ in an important point. Statistical laws are bound to huge ensembles they describe physical systems consisting of a huge number of objects. The greater the number of objects the more strict is the law about the whole ensemble. Though there is indeterminacy for every individual system, there is a strict law for the whole system if the ensemble is large enough. To some extent such laws "emerge" from the "lawless" behaviour of a large number of individual systems. In this sense Wheeler spoke of "law without law"[14]. This description fits very well to the statistical laws in Thermodynamics. Concerning the statistical laws in QM there is an additional problem: though it is clear also here that the theory refers to big ensembles of prepared systems the question is whether this is the only reference; i.e. whether it also refers to individual quantum systems like singular photons or electrons. This question arises especially in connection with series of new experiments[15].

Despite of this complication in QM, it holds for all important statistical laws that the individual system is not definitely described by the law but has its degrees of freedom which are not restricted by the law. This shows unambiguously the difference between D2 and S2. And it shows again that w.r.t. D2 and S2 neither type of law is reducible to the other.

(iii) The difference between dynamical and statistical laws which is usually viewed as the striking difference is that which is expressed in D3 and S3: Dynamical laws are invariant under time-reversal, statistical laws are not. The former describe processes which are time (reversal) symmetric, the latter describe processes which are irreversible;

But this difference has been weakened within the last years to a considerable extent. First indirect violations of time-reversibility have been found, recently also direct ones. Concerning the indirect violations it is known already since decades that CP (charge-parity)

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[13] This is however not the place to enter a discussion about the ontological status of statistical laws.

[14] Wheeler (1980) p. 363. Cf. The quotation of Schrцdinger above.

[15] These are mainly so-called Split-beam experiments with photons and other particles and experiments which hold particles in a „cave". For the emergence of statistical laws in QM see Mittelstaedt (1997).

is violated in weak interaction with neutral K-mesons. Since CPT (the combined symmetry of charge parity and time) – one of the most important symmetries of Quantum Field Theory – seem to be universally satisfied T has to outbalance the difference and therefore T-reversibility cannot hold unrestrictedly. Concerning the direct violation there have been two different series of experiments independently made in CERN and FERMILAB which proof the violation: The time dependent rates for the strangeness-oscillation process from K^0 to $\overline{K^0}$ and its inverse from $\overline{K^0}$ to K^0 (neutral kaons) are different (CERN)[16]. If time-reversal symmetry were strictly preserved we should have identical rates. Since T, unlike P, reverses also the spin of the particle an angular variable φ was measured for each decay. Time reversal symmetry would require that the φ (and sin 2 φ) distributions are symmetrical about zero. The observed asymmetry is about 14 % which agrees with the theoretical expectation (FERMILAB)[17].

Concerning D4 and S4 it holds that there is a considerable difference if D4 is satisfied. Then the system governed by dynamical laws relaxes if it is disturbed in a modest way. In this case the predictability also of singular parts of the system (singular objects) does not decrease. If however D4 is not or only partially satisfied then we have (stronger or weaker) chaotic motion. In this case there is no predictability – although the system is governed by dynamical laws – and loss of information about the diverging parts of the system according to the Kolmogorov entropy (see level six below). In this case there is a certain similarity to S4 w.r.t. the individual objects or small parts of the system: in both cases there is an exponentially growing loss of information about the individual parts of the system. But this similarity should not mislead: In the disturbed case of a dynamical system its chaotic behaviour does not even satisfy statistical laws[18], or satisfies only partially statistical laws in the following sense: In some cases one can take into account ensembles of trajectories instead of single trajectories; since single trajectories which differ very little w.r.t. their initial conditions (their starts) diverge exponentially in the course of time. Whether such a description is possible depends on the degree of how chaotic a motion is[19].

[16] Angelopoulos et al. (1998).
[17] Cf. Schwarzschild (1999)
[18] An example is the chaotic pendulum. For a description see Lighthill (1986).
[19] Chirikov (1991)

4.2 Statistical laws are compatible with dynamical laws

(i) Concerning D1 and S1 both are easily compatible w.r.t. to huge ensembles of individual objects. In this case also the statistical law can make definite statements about the future state. The individual state however is not determined by the earlier state in processes of thermodynamics, radiation, friction etc. (provided we accept a realistic interpretation of statistical laws, i.e. assuming real degrees of freedom of the individual object in such phenomena).

(ii) Zermelo thought that he had proved that dynamical laws and statistical laws (like Boltzmann's law of entropy) are incompatible. He used Poincarй's recurrence theorem to show that Boltzmann's statistical mechanics cannot be correct[20]. However, as the replies of Boltmann show[21] Zermelo partially neglected and partially misunderstood important conditions in connection of Poincarй's recurrence theorem. Zermelo did not realize under which physical conditions this theorem is not applicable and that the recurrence depends very much on the complexity of the system; such that with increasing complexity the probability of the recurrence of the state of the system becomes extremely low[22].

(iii) At the time of Zermelo there was of course a strong belief that dynamical laws describe processes which are time-reversible. As it was said in 4.1(iii) time reversal symmetry does not strictly hold on the micro level. On the other hand Boltzmann was always very modest concerning claims of "irreversibility". Although he used this term he immediately added that it means that the probability of the recurrence of the whole system is very low and decreases exponentially with the complexity of the system. Taken in this interpretation there is no incompatability.

(iv) The compatibility of dynamical and statistical laws even within one physical system is illustrated in a lucid way by a *Gedankenexperiment* of Lee[23]:

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[20] Zermelo (1896a,b)

[21] Boltzmann (1896) and (1897).

[22] Zermelo was at that time assistant to Max Planck who didn't go so far in his views as his assistant. For that see a letter from Planck to Graetz, cited in Kuhn (1978) p. 27. Cf. also Weingartner (1999) After Boltzmann's criticism of Zermelo's alleged proof, Zermelo left physics and worked on the foundations of mathematics (Set theory) where he became very successful. However, he still behaved stubborn w.r.t. to new discoveries or proofs. In October 1931 he wrote to a friend that Gцdel's proof (of the incompleteness and undecidability of arithmetic) is nonsense. See the letter published in Weingartner-Schmetterer (1987) p. 45 f.

[23] Lee (1988) p. 16 f.

Assume a number of airports with flight connections in such a way that between any two of these airports the number of flights going both ways along any route is the same. This property will stand for microscopic reversibility. Some of the airports may have more than one air connection (they are connected with more than one other airport) whereas other airports have a connection only to one airport (let's call such airports dead end airports). A passenger starting from a dead end airport (or starting from any other airport) can reach any other airport and can also get back to his starting airport with the same ease. This property stands for macroscopic reversibility. In this case we have both mircroscopic and macroscopic reversibility.

But suppose now we were to remove in every airport all the signs and flight informations, while maintaining exactly the same number of flights. A passenger starting from a dead end airport A will certainly reach the next airport B since that is the only airport connected with A. But then especially when assuming that B has many flight connections – it will be very difficult to get further to his final destination, in fact it will be a matter of chance. Moreover his chance to find back to his dead end airport A will be very small indeed. Thus in this case we have microscopic reversibility maintained but macroscopic irreversibility or very improbable recurrence – especially if we think of millions of passengers flying around randomly - and both are not in conflict.

Fifth Level: Laws and Causality

One level or kind of understanding of laws of nature which has been of great importance in the whole history of philosophy and of science is connected with causality. And after the more accurate interpretation of dynamical law with the help of differential equations (from Newton's time on) this type of causal relation got a very definite interpretation: In the time development of a physical system its state S_1 (which corresponds to a solution of the differential equation) is the cause of its later state S_2 (which corresponds again to a solution of the equation) where the causal relation between cause S_1 and effect S_2 is represented by the law of nature formulated by the differential equation. This was an exact interpretation of one type of cause of the four traditional causes described by Aristotle in his metaphysics[24]. The corresponding understanding of laws of nature was such that every law

[24] Aristotle (Met) book V, ch. 2. This type of cause fits probably best to the so called causa efficiens.

of nature represents a causal relation or describes a causal connection which is such that the following conditions are satisfied:

(i) It satisfies D1 and D2, i.e. the causal relation takes place not only between the earlier and later states of the system but also between all its sub-systems, ultimately between the individual objects or particles. This condition, i.e. a causal relation according to D1 and D2 is often expressed in the following way:

CD The same initial state leads – under the same conditions – to the same series of successor states.

(ii) Every causal relation represented by laws of nature and describing causal processes is spacio temporal continuous.

(iii) It satisfies D4, i.e. it presupposes physical systems with a high degree of stability such that perturbations will not destroy definite predictability of the effects.

Processes like those of thermodynamics didn't satisfy (i), those of Quantum Mechanics (QM) didn't satisfy (ii) and those of chaotic motion didn't satisfy (iii). Therefore the natural question was whether such processes are causal.

The first question is whether there is some kind of causality represented by statistical laws. Such a kind of causality cannot satisfy D1 and D2. Also condition (ii) should not be a necessary condition for such a kind of causality. A causal relation for statistical laws which is in accordance with S1-S4 was proposed by March:[25]

CS The same initial state may lead to different successor states. But those successor states which belong to the same initial state obey the same statistics.

The second question is whether there is some kind of causality which can be accepted in quantum mechanical processes. As is known Heisenberg thought that the uncertainty relations prove that there cannot be "Since all experiments have to obey the equation $\Delta p \cdot \Delta q \geq h$ viz. $\Delta W \cdot \Delta t \geq h$ QM establishes finally the invalidity of the causal law"[26]. Concerning this second question one should not forget first that D1 plus D4 are valid not only for objects which can be treated classically (described by Hamilton equations) but also for those QM-objects to which definite properties can be attributed by the Schrцdinger equation. Secondly one should notice that the constraints of the uncertainty relations do not forbid every causal relation but only those which presuppose a principle (FS) which was indeed a hidden assumption in classical physics and also in the philosophical tradition:

[25] March (1957) p. 14

[26] Heisenberg (1927) p. 197

FS Any two quantities (out) of all observables can be measured instantaneously to a suitable degree of exactness; or more accurately in such a way that the values of measuring are free of dispersion and determined by hidden parameters which are themselves not observable.

The principle FS is not satisfied in QM. However, QM is not the only area where this or analogous principles are not satisfied. There is a more general principle which belongs to the formation rules underlying classical logic. It says that every proposition can be combined with any other one to form a new proposition and every predicate can be combined with any other one to form a new (complex) predicate. One can easily see that these principles have to be taken with care when logic (presupposing them) is applied to certain fields: Since p, q…etc. are variables they can be instantiated by any statement about empirical events. Thus if p represents the occurrence of human action h_1 and q represents the occurrence of human action

h_2 then it is not always the case that $p \wedge q$ represents also a complex action h_3. Thus the events described cannot be combined like the describing propositions. The following example form zoology is concerned with incompatible predicates: Sexual excitement and fear cannot be observed (or measured) at the same time in male animals but it can in female animals while sexual excitement and aggression cannot be observed a the same time in female ones but in male animals. These examples show that the invalidity of such or similar principles like FS is not a speciality of QM but is rather general and is distributed among different fields of reality which are investigated by scientific research. The conclusion of all that is simply that any concept of causality which implies (or presupposes as a necessary condition) the principle FS is not suitable for QM and also not for other areas of natural science.

The third question is whether there is some kind of causality which can be accepted in processes of chaotic motion. To this one may answer that in the case of dynamical (or deterministic) chaos the underlying laws are dynamical laws and therefore nothing hinders to apply here a concept of causality which is that of dynamical laws i.e. satisfies D1, D2, (i) and (ii) though does not satisfy D4. That it does not satisfy D4 means that causality should not be confused with predictability. Thus though D1 and D2 are satisfied (i.e. there are dynamical or deterministic laws) chaotic motion is not predicable except for extremely short time intervals.

Sixth Level: Chaotic Motion

The discovery of chaotic motion showed something new which was not understood in connection with laws of nature so far. It showed that initial conditions, boundary conditions or mathematical proportions are not necessarily accidental but can play an important role w.r.t. several properties of the laws. The new discovery was that even within the area of relatively simple physical systems which perfectly obey dynamical laws of Classical Mechanics, like the spherical pendulum, such systems can change radically its behaviour. Thus a dynamical system obeying Newton's laws with strict predictability can become chaotic in its behaviour and practically unpredictable just by changing slightly some initial conditions. Experiments which prove such a behaviour of dynamical systems have been made since the seventies. A special kind of very simple arrangements are experiments with the so-called forced pendulum or with the kicked rotator. Such systems satisfy normally conditions D1 to D4 (cf. chapter 3 above). But a small change in the initial conditions force the system to violate D3 and D4. Concerning the spherical pendulum the important new discovery is now that this simple physical system becomes chaotic if the top end is forced to move back and fort (maximal displacement Δ) with a slightly different period T greater than T_0, provided that Δ is about 1/64 of l and not more than about a tenth of the energy of motion is dissipated by damping (air resistance etc.). Miles (1984) showed experimentally that the system is chaotic for values of $T = 1{,}00234T_0$. It has to be emphasized however that this does not just mean that the system becomes unstable in the sense of simple bifurcation. In this case – and this was known for a long time – the pendulum will relax after a certain time. But for the values above the pendulum is breaking out of the plane, the bifurcations are increasing the dependence on the initial conditions is completely random and there is no predictability. Nobody knows why this happens exactly at $T = 1{,}00234T_0$. And this shows that there is still a lot of ignorance concerning special values of initial conditions and in general concerning their role w.r.t. to the laws. The important necessary condition for chaotic behaviour is the sensitive dependence on the initial condition; or the fact that small changes in the initial states lead to exponentially increasing changes in the course of the time development.

Historically it is interesting that already Aristotle had some understanding of such an exponential growth:

"…the least initial deviation from the truth is multiplied later a thousandfold"[27].

A modern interpretation of Aristotle's observation of increasing error is given by the so-called Hйnon-attractor[28]. The error increases exponentially with a factor a^t , where a ˜ 1,52 and t is a time unit. A deeper understanding is shown by Maxwell in the following quotation:

"There is another maxim which must not be confounded with that quoted at the beginning of this article[29], which asserts 'That like causes produce like effects'. This is only true when small variations in the initial circumstances produce only small variations in the final state of the system. In a great many physical phenomena this condition is satisfied; but there are other cases in which a small initial variation may produce a very great change in the final state of the system, as when the displacement of the "points" causes a railway train to run into another instead of keeping its proper course"[30]. The example of Maxwell shows that the sensitive dependence on initial conditions is not a defining condition (necessary and sufficient) for chaotic motion but only a necessary, though very important one: The unproportional effect need not to be chaotic (as the running of the train into a different direction); however the crash may be partially a chaotic phenomenon. On the other hand Newton though considering weak perturbation didn't pay much attention to such initial conditions as for example the distances of the planets and their mathematical proportions. In contradistinction Kepler was convinced that these proportions (which he calculated to be approximate to the golden cut) are necessary for the harmony (in today's terms: stability) of the planetary system. This is not the place to go into details about chaotic motion. Elsewhere I have given eight necessary conditions for dynamical chaos[31]. The new message however is, w.r.t. the question of understanding what a law (of nature) is, that initial conditions, boundary conditions and mathematical proportions are much more important than they seemed to be until the second half of the last century.

Seventh Level: Initial Conditions and Constants of Nature

The understanding of what a law is depends on the distinction between laws and initial conditions. This goes back to the Greeks (see

27 Aristotle (Heav) 271b8
28 Hйnon (1976)
29 The one to which Maxwell refers is „The same causes will always produce the same effects" which he discusses earlier.
30 Maxwell (1992) p.13
31 Weingartner (1996) p. 52 f.

first level of understanding). The deeper problems of such a distinction are very well described by the following quotations of Wigner and Wheeler:

"The world is very complicated and it is clearly impossible for the human mind to understand it completely. Man has therefore devised an artifice which permits the complicated nature of the world to be blamed on something which is called accidental and thus permits him to abstract a domain in which simple laws can be found. The complications are called initial conditions; the domains of regularities, laws of nature. … The artificial nature of the division of information into "initial conditions" and "laws of nature" is perhaps most evident in the realm of cosmology. Equations of motion which purport to be able to predict the future of a universe form an arbitrary present state clearly cannot have an empirical basis. It is, in fact, impossible to adduce reasons against the assumption that the laws of nature would be different even in small domains if the universe had a radically different structure. One cannot help agreeing to a certain degree with E.A. Milne, who reminds us that, according to Mach, the laws of nature are a consequence of the contents of the universe. The remarkable fact is that this point of view could be so successfully disregarded and that the distinction between initial conditions and laws of nature has proved so fruitful[32].

The question what fixes the initial conditions was already discussed in the 13th century at this very university (Sorbonne, Paris) where I am giving my talk now: The question about which Thomas Aquinas and Bonaventura had a fight was that of the initial conditions of the universe. Can it be proved rigorously (i.e. in the sense of a *demonstratio* which is based on laws) that the world had a beginning (and has a finite age)? Bonaventura thought he can prove that by showing that so far not infinitely many states of the universe could have been past. Thomas Aquinas defended the view that this cannot be proved with the help of laws about this world (universe). And his argument was very simple: genuine laws abstract from hic (here, place) and nunc (now, point of time). Therefore we cannot derive any singularity (initial condition) form a law[33]. This points to a certain incompleteness of all genuine laws and of all laws of nature. They do not fix certain initial conditions. Form this it follows that initial conditions can be changed without changing laws. Thus our laws must be valid also in other possible universes which differ from ours only with respect to initial conditions. This way of thought was used by Popper to

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[32] Wigner (1967) p. 3. Milne (1948) p. 4.
[33] Cf. Thomas Aquinas (STh) I, 46,2.

define the kind of necessity which pertains to laws of nature, natural or physical necessity: "A statement may be said to be naturally or physically necessary iff it is deducible from a statement function which is satisfied in all worlds that differ from our world (if at all) only with respect initial conditions"[34]. But the question what kinds of initial conditions can change without changing the laws of nature is a very difficult one. It leads to the general question of the set of all those changes which do not change laws (of nature). This set Weinberg called the symmetry group of nature about which he wrote:

"It is increasingly clear that the symmetry group of nature is the deepest thing that we understand about nature today…Specifying the symmetry group of nature may be all we need to say about the physical world beyond the principles of quantum mechanics"[35]. That the laws of nature are valid not just in our universe but also in others which differ from ours only w.r.t. some special initial conditions I have defended elsewhere[36]. There are two main reasons for that: (1) not all laws of nature are deterministic dynamical laws but some are statistical laws which allow branching and degrees of freedom for the individual particle. (2) It is impossible that all initial conditions (for example micro-states) which are compatible with all the laws of nature occur as states (are played through) during the life time of our universe, provided this life time is finite.

Concerning constants there is the difficult question whether the constants of nature are really constant. The important constants for non-relativistic Quantum Mechanics (QM) are h (Plancks constant), m_e (mass of electron) and e (elementary charge). For relativistic QM the three main constants are h, c (light velocity) and G (gravitational constant). In addition there are the dimenson-less constants α (Fine structure constant)and the proportion of proton mass to electron mass (1836). If at least one of these constants would change the fundamental laws (in which they occur) would not be time(translation)invariant. Though there is intensive research done in order to be able to rigorously test whether some of these constants change (slowly) with time, there is not a clear experimental evidence for it so far. Another question connected with this one is the explanation or the deeper reason for these magnitudes. With this question Dirac was concerned from 1937 on. In his last paper on that in 1973 he writes.

"At present, we do not know why they should have the values they have, but still one feels that there must be some explanation for them

[34] Popper (1959) p. 433.
[35] Weinberg (1987) p. 73
[36] Cf. Weingartner (1996) ch. 7.2

and when our science is developed sufficiently, we shall be able to calculate them"[37]. There he formulates also his *Large Numbers Hypothesis*. This hypothesis states that very large numbers cannot occur without reason in the basic laws of physics.

"It involves the fundamental assumption that these enormous numbers are connected with each other. The assumption should be extended to assert that, whenever we have an enormous number turning up in nature, it should be connected to the epoch and should, therefore, vary as t varies. I will call this the Large Numbers hypothesis[38]. One of Dirac's examples for an equation satisfying his *Large Numbers Hypothesis* was: $e^2/G\ ?\ m_e \cdot m_p = T/t_e$ (where the left part is the ratio of electric force and the gravitational force of electron and proton and the right part is the ratio of the age of the universe and the time the light needs for the diameter of the electron). Both magnitudes are dimensonless and of the order 10^{40}. If this equation is true then the laws of nature are not time(translation)symmetric since G would decrease with time. Applied to the Big Bang Theory the *Large Numbers Hypothesis* implies continuous creation of matter which would violate the law of conservation of energy. However, there is not enough experimental evidence for a decision concerning these questions. But independently of that we may say that even if the laws of nature would change very slowly in accordance with very slow changes of the fundamental constants due to the development of the universe such laws of nature would be invariant (symmetric) enough to both (i) justify the distinction between initial conditions and constants on one hand and laws on the other and (ii) explain contingent facts with the help of these laws.

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[37] Dirac (1973) P. 45

[38] Ibid. P. 46

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